Journal of Economic Behavior and Organization 195 (2022) 335–358

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Fixed price equilibria on peer-to-peer platforms: Lessons from time-based currencies☆

Berno Buechel[a,*], Philemon Krähenmann[b]

[a] University of Fribourg, Department of Economics, Bd. de Pérolles 90, Fribourg CH-1700, Switzerland
[b] University of St. Gallen, Institute of Economics (FGN), Varnbüelstrasse 19, St. Gallen 9000, Switzerland

A R T I C L E I N F O

Article history:
Received 1 July 2020
Revised 21 December 2021
Accepted 12 January 2022
Available online 11 February 2022

JEL classification:
D51
D61
D63
L14

Keywords:
Platforms
P2P
Non-competitive prices
Welfare
Walrasian equilibrium
Networks

A B S T R A C T

There are many online platforms for peer-to-peer exchange that introduce a platform-specific currency and fix prices to some extent. We model such platforms as pure exchange economies and characterize all fixed price equilibria. We demonstrate the inherent inefficiency following from the combination of fixed prices and voluntary trade and show that simple additional Pareto improving trades exist. While our theoretical analysis predicts that fixed prices lead to less trade than competitive prices, we also discuss some potential advantages of price restrictions, in particular, with respect to simplicity and fairness considerations. Strikingly, it is not those agents with the most attractive goods who have the highest willingness to participate in a fixed price platform. In an empirical illustration, we explore how these theoretical insights unfold in reality by describing patterns of several platforms covering around 100k transactions. This work is informative for the market design of peer-to-peer platforms and for markets with price restrictions more generally.

1. Introduction

Among the many online platforms that have emerged in recent years, an interesting kind are platforms for peer-to-peer exchange with a platform-specific currency. Members can trade there as on conventional marketplaces, except that one cannot solely be a buyer or a seller. Instead, every participant must be to some extent a buyer and a seller.[1] This

☆ We thank Juan Beccuti, Jean-Jacques Herings, Joao Montez, Daniel Habermacher, Bodo Hilgers, Steven Tadelis, Armin Schmutzler, Walter Trockel and the audiences of the virtual World Congress of the Econometric Society 2020, the Swiss IO Day in Bern, the Swiss Theory Day in Zurich, the European Meeting of the Econometric Society in Manchester (ESEM 2019), as well as the seminar participants in Bendern and Maastricht for helpful comments. This work has moreover greatly benefitted from the comments by Stefan Buehler, Stefan Kloessner and three anonymous reviewers. Special thanks go to Richard Faltings and Bénédicte Droz for excellent research assistance. Berno Buechel gratefully acknowledges research support by the Swiss National Bank (SNB) provided to the Department of Economics of the University of Fribourg. Philemon Kraehenmann thanks the Swiss National Science Foundation (SNSF) for generous support (P1SGP1_181676).

* Corresponding author.
E-mail addresses: berno.buechel@unifr.ch (B. Buechel), philemon.kraehenmann@unisg.ch (P. Krähenmann).
URL: http://www.berno.info (B. Buechel)

[1] In a broader definition of "peer-to-peer platforms," buyers need not be sellers and sellers need not be buyers. Einav et al. (2016) discuss such platforms and compare them to traditional markets.

https://doi.org/10.1016/j.jebo.2022.01.013
0167-2681/

feature is typically guaranteed by a platform-specific currency that can be earned only through sales on the platform and that can be used only for purchases on it. There are different reasons why a platform operator might want to create such a closed exchange marketplace. First, because it provides incentives for interested buyers to also contribute as a seller. Second, because it commits sellers to spend their earnings among the participants.[2] Third, it is a way to differentiate themselves from other platforms, e.g. some platforms exclude professional sellers, which might also serve to avoid certain regulations. Finally, issuing a platform-specific currency can serve as a way of funding the platform's business.[3] Examples for platforms of peer-to-peer exchange include guestoguest.com, where members can rent homes with guestpoints. These points can only be earned by renting one's own home to other members, while the maximal price one is allowed to charge depends on defined house characteristics. On bookmooch.com members can swap books, where each book costs exactly one point. Further, so-called time banks allow for local exchange of services, where one hour of service is typically fixed to cost one hour of a time currency. As these examples show, many of these platforms restrict price setting. Their motivation to do so could be to guarantee some price stability on the platform, to increase market transparency, or for some kind of fairness considerations.[4] However, the consequences of the platform operator's decision to keep prices rather fixed or rather flexible are not well understood.

In this paper we model such marketplaces and study the effect of price setting restrictions on efficiency, extent of trade, and the incomes. We believe this is interesting for at least two reasons. First, platforms for peer-to-peer exchange have recently popped up all around the world and for various kinds of goods. Even though marketplaces where members have to be active on the demand and supply side have existed at least since the nineteenth century (see e.g. Warren, 1852), creating such systems has become much more common when internet lowered transaction costs.[5] Despite the creation of many peer-to-peer platforms, their popularity lags tremendously behind online platforms such as eBay, Amazon or Alibaba, which feature flexible prices and where sellers need not be buyers. Shedding light on the workings of such platforms is informative for their market design. In particular, rules of price setting, which we can study with our framework, seem to be a crucial feature of a platform's market design.

Second, economists have been interested in general equilibrium effects in closed exchange economies for a long time. Peer-to-peer exchange platforms are wonderful real-world examples for such closed exchange economies. Hence, we can make use of a rich body of theoretical work, in particular, on the properties of equilibrium allocations with and without Walrasian prices, and link this theoretical work to recent empirical observations.

We model a simple exchange economy with fixed prices. Each agent can offer his endowment and consume goods that are offered by others. Goods can only be traded for a platform-specific currency. To keep the model simple, agents are assumed to have additively separable preferences, which are quasi-linear in the currency and strictly convex.[6] We look for fixed price equilibria. The corresponding equilibrium concept was introduced by Drèze (1975) and is often referred to as Drèze equlibrium. We use the formulation of Maskin and Tirole (1984), whose "K-equilibrium" coincides with the Drèze equilibrium in our setting. Maskin and Tirole (1984) show that a fixed price equilibrium naturally incorporates two properties: First, no agent can be forced to trade ("voluntariness") and, second, there is no pair of agents who can improve by trading some good ("weak order"). When the fixed prices happen to coincide with Walrasian equilibrium prices, then the fixed price equilibrium and the Walrasian equilibrium allocations coincide. Otherwise, fixed prices necessitate that some agents are constrained from buying or from selling certain goods.

Assuming quasi-linearity of preferences allows us to characterize all fixed price equilibria and to derive empirical predictions about the effect of price setting restrictions in these markets. The starting point of our analysis is the distinction between *scarce* goods and *non-scarce* goods. The former ones are goods for which market demand at given prices is larger than the total endowment. For non-scarce goods market demand is smaller. We show that in any fixed price equilibrium, sellers providing a scarce good keep their optimal amount of that particular good (while all buyers receive at most their desired amount). Further, all buyers receive their optimal amount of each non-scarce good, while the seller of the non-scarce good keeps the rest, which is more than this agent desired. In other words, the seller of a non-scarce good is constrained from selling the desired amount, and at least one of the buyers of a scarce good is constrained from buying the desired amount. The rationing scheme therefore only affects the allocation of scarce goods, but not the allocation of non-scarce goods, which must be the same in every fixed price equilibrium (when every agent offers only one good).

The first implication of this characterization is that, under very weak conditions any fixed price equilibrium is not only Pareto *in*efficient, but also *constrained inefficient*. Indeed, we can construct simple chains of bilateral trades that are Pareto improvements within the given price system, under weak conditions on the existence of either strictly scarce or strictly non-scarce goods. Thereby, each bilateral trade either involves an agent who is constrained seller of a non-scarce good and can sell more of his good, or constrained buyer of a scarce good who can buy more of this good. In the simplest case, there are two suppliers of non-scarce goods who have a non-zero demand for each other's good. Then they can both improve by

2 See Mailath et al. (2016) for a formalization of that argument.

3 See, e.g. Bakos and Halaburda (2019) for a discussion of this funding approach.

4 We will discuss the reasons to keep prices fixed in more detail in Section 6, i.e. when we can relate to our results.

5 When Josiah Warren promoted time-dependent currencies in his "Cincinnati Time Store" in the years 1827–1829, little did he know that this idea had to wait for the rise of the internet to become picked up much more frequently. Still, however, the created systems only serve a niche market.

6 We tailor the assumptions to the application and keep the model simple. This buys us clear-cut results that make the underlying effects transparent. We study robustness to relaxing the assumptions in Appendix B.

exchanging their services. However, in a market with fixed prices this will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In that sense, the price of their goods is "too high." The case with "too low" prices works similarly, and there are also combinations of the two.

We then proceed by comparing fixed price equilibria with Walrasian equilibria, which are a special case of fixed prices, but can also result under flexible prices when prices are determined competitively. It turns out that the extent of trade in the Walrasian equilibrium is larger than in any other fixed price equilibrium. That is, every agent can sell weakly less in a fixed price equilibrium than in the Walrasian equilibrium. In the generic case that each good is strictly scarce or strictly non-scarce, in any fixed price equilibrium the amount traded of any good is even strictly smaller than in the Walrasian equilibrium. Hence, it becomes apparent that fixed prices hamper trade, which is a clear downside of most such platforms. However, Walrasian equilibria do not Pareto dominate fixed price equilibria in general, with the consequence that both regimes generate their "winners" and "losers." The winners of flexible prices are typically suppliers of scarce goods because they sell more and at a higher price, which boosts their income. The winners of fixed prices are typically suppliers of non-scarce goods for whom the fixed price happens to be close to the market clearing price, as they have the highest willingness to pay to participate in the platform. We finally investigate data from seven time exchange markets, covering almost 100,000 transactions. These are peer-to-peer exchange platforms, facilitating decentral trade typically through a time-based currency. Prices are fixed to different degrees. We observe that those platforms with fixed prices indeed have less transactions than those with rather flexible prices and that their trade networks are less dense and more equal.

Our paper makes three contributions. First, we show that price restrictions, which are a common feature of peer-to-peer platforms with a platform-specific currency, come at a very high cost. We show theoretically and illustrate empirically that under fixed prices and decentral trade participants leave out Pareto improving trades, even within the given price regime. The relatively low number of transactions and the correspondingly low number of customer-supplier relationships indicate that price restrictions seriously hamper the working of the market. This contrasts with centralized markets in the tradition of Shapley and Scarf (1974)'s housing market, where appropriate matching mechanisms can often eliminate such inefficiency problems (e.g. Abdulkadiroglu and Sönmez, 1999). In particular, Andersson et al. (2021) propose a matching mechanism for time exchange markets that is able to maximize the traded time and is strategy-proof on a domain of preferences that suits this application.

Second, we study advantages and disadvantages of price restrictions in the market design of peer-to-peer platforms. Theoretically, fixed prices that are perfectly set can reach any goal (profit, welfare, equality, ...) at least as good as flexible prices would. Practically, platforms face a trade-off between inefficiency due to heuristically set fixed prices and inefficiency due to monopoly pricing under price flexibility. Moreover, we find that relaxing price restrictions has unequal effects on the market participants. This might lead to a different selection of who joins the platform in the first place. In particular, agents with the most attractive goods have weak incentives to join the fixed price platforms, a finding that seems completely novel in the literature. We then propose different alternatives in the market design that incorporate several concerns of the market participants.

Third, we apply general equilibrium theory, in particular on Walrasian and fixed prices in exchange economies, to a new setting and derive predictions that could be empirically tested. It is well known that non-Walrasian market allocations are generally not Pareto efficient. Moreover, it has been shown that such allocations typically do not even satisfy *constrained efficiency*, that is, there exist Pareto improving trades within the given, non-Walrasian price regime (Younés, 1975; Maskin and Tirole, 1984; Herings and Konovalov, 2009). We do not only show for the application of peer-to-peer platforms that this insight applies, but we characterize the inefficiency more specifically by showing how "too high" or "too low" prices prevent some simple Pareto improving trades. In comparison to Herings and Konovalov (2009), we make more simplifying assumptions on the utility functions of the market participants, but stay more general in terms of admitting boundary solutions and not imposing a particular rationing scheme. We think that in our application and in many others it is an important feature that a given participant need not buy all products that are in the market.

We think that our results are also informative for market design outside of peer-to-peer platforms. In many real-world markets prices are (at least in the short run) non-Walrasian. There are several causes of price stickiness, such as costs of changing marketing activities, consumers' perceptions of clear or "fair" prices, or governmental regulations. Our analysis of closed exchange economies suggests that on many more markets price restrictions hamper trade, induce an inefficient allocation, and have some desired benefits that were also attainable with a variation of the market design.

In the next section, we introduce the model. Section 3 presents the main results. Section 4 addresses the platform's optimization problem. The empirical illustration follows in Section 5. In Section 6 we discuss advantages and disadvantages of peer-to-peer exchange platforms with fixed prices, before we conclude in Section 7. The appendix contains proofs, extensions, and details about the empirical illustration.

2. Model

Consider a pure exchange economy with $n \geq 2$ agents, $N = \{1, 2, \ldots, n\}$, indexed by i; and $m+1$ goods indexed by h ($h = 0, 1, \ldots, m$). A price vector $p \in \mathbb{R}^{m+1}$ with $p_0 = 1$ and $p_h > 0$ is exogenously fixed. Each agent i has consumption set $X^i = \mathbb{R} \times \mathbb{R}^m_+$ and is characterized by the endowment $\omega^i \in X^i$. Each agent i has complete and transitive preferences $\succsim^i$ over consumption bundles X^i, represented by a utility function $U^i : X^i \to \mathbb{R}_+$. We assume that preferences are continuous and strictly convex.

For the main part of our analysis, we make the simplifying assumption that each agent i is endowed with exactly one good such that $\omega^i_i > 0$, while $\omega^i_h = 0$ for $h \neq i$ and $m = n$. This assumption is tailored to the examples of service exchange and house exchange, while the arguments extend to the more general case.[7]

For the application of service exchange with a time-based currency, which is used in the empirical illustration in Section 5, we consider the following interpretation of the model. Each agent j can provide one service $h = j$. A service j is quantified by the amount of time agent j needs to provide that service. Let x^i_j be the amount of time that j stands in the service of i. We denote by $u^i_j(x^i_j)$ the utility agent i derives from service of agent j.[8] Services are priced on that basis. Each hour of service costs one amount of the numeraire good $h = 0$, so $p \equiv (1, \ldots, 1)$. The numeraire good is not a service but a time-based currency. Agents are not initially endowed with the time-based currency but can hold negative amounts of it, e.g. to initialize trade. The utility from consuming the own service, i.e. $u^i_i(x^i_i)$, measures the opportunity costs of agent i from providing his service. This could be, for example, the indirect utility from the income when investing the amount of time x^i_i in a work outside of the platform. The utility agent i derives from the time currency is $u^i_0(x^i_0)$. We can interpret this as the indirect utility from using time currency for consumption in a later period. If the opportunity exists to convert the numeraire good into money, we can alternatively interpret it as the indirect utility from the amount of money one can convert the amount x^i_0 into.[9]

We focus on preferences that are additively separable and quasi-linear in the numeraire.[10] Utility of agent i is given by:

$$U^i(x^i) = x^i_0 + u^i_1(x^i_1) + \ldots + u^i_i(x^i_i) + \ldots + u^i_n(x^i_n).$$

We assume that u^i_h is twice differentiable with marginal utility $mu^i_h(x^i_h) > 0$, $\frac{\partial mu^i_h(x^i_h)}{\partial x^i_h} < 0$ and $\lim_{x^i_h \to \infty} mu^i_h(x^i_h) = 0$ for all $i, h \neq 0$ and x^i_h.

Let us now turn to the equilibrium concept. As is well-known, for fixed prices we can in general not expect the feature of Walrasian equilibrium that individual optimal decisions are consistent with market clearing. Instead some agents are constrained from selling or from buying on certain markets. The corresponding equilibrium concepts for fixed prices (i.e. in general non-Walrasian prices) are based on two fundamental principles:

(i) *voluntariness*: no agent can be forced to trade. (Otherwise, his choice could be inconsistent with his preferences.)
(ii) *weak order*: no two agents can be constrained on two different sides of the same market. (Otherwise, they could improve by trading.)

In our setting, several prominent equilibrium concepts coincide. While we apply the common Drèze equilibrium (Drèze, 1975), we precisely follow Maskin and Tirole (1984) by defining a fixed price equilibrium based on the two principles above. For this purpose, we need some additional notation. Agent i's consumption bundle $x^i \in X^i$, can be captured by his net trades t^i:

$$x^i = \omega^i + t^i$$

and likewise we can construct a set of trades $T^i = \{x^i - \omega^i | x^i \in X^i\}$ of agent i. Since in our context there is only one seller on each market, the endowments are of the form $\omega^i = (0, \ldots, 0, \omega^i_i, 0, \ldots, 0)$. For all $i \neq h$ we therefore have $x^i_h = t^i_h$ with non-negative t^i_h when $h \neq 0$; and for $i = h$ we have $x^i_i = \omega^i_i + t^i_i$ with $t^i_i \geq -\omega^i_i$. Let $\tilde{T}^i := T^i \cap \{t^i | p \cdot t^i = 0\}$ be the set of (with respect to budget) feasible net trades of agent i. We define $\tau^i_h(t^i) := \{\tilde{t}^i \in \tilde{T}^i | \tilde{t}^i_k = t^i_k \ \ \forall k \neq 0, h\}$, as the feasible net trades of agent i that coincide with the net trades t^i on all markets, but on market h and 0. We denote by $T = \{(t^1, \ldots, t^n) \in (T^i, \ldots, T^n) | \sum_i t^i = 0\}$ the set of (mutually compatible) net trades in the economy and by $\tilde{T} = \{(t^1, \ldots, t^n) \in (\tilde{T}^i, \ldots, \tilde{T}^n) | \sum_i t^i = 0\}$ the set of (mutually compatible and with respect to budget) feasible net trades in the economy. Finally, let $Z = ((\underline{Z}^1, \bar{Z}^1), \ldots, (\underline{Z}^n, \bar{Z}^n))$ be a vector of quantity constraints such that $\underline{Z}^i \leq 0$ and $\bar{Z}^i \geq 0$ and $\underline{Z}^i_0 = -\infty$ and $\bar{Z}^i_0 = \infty$ for all i.[11]

We can now define equilibrium allocations x under fixed prices p by defining the corresponding equilibrium trades t.

Definition 1 (Fixed Price Equilibrium, Maskin and Tirole, 1984)**.** A fixed price equilibrium (FPE) is a vector of feasible net trades $t \in \tilde{T}$ associated with a vector of quantity constraints Z such that for all i,

(V) exchange is "voluntary:" t^i is the $\succsim^i$-maximal element among the (budget) feasible net trades $\tilde{t}^i \in \tilde{T}^i$ that satisfy the constraints $\underline{Z}^i \leq \tilde{t}^i \leq \bar{Z}^i$.

---

[7] We relax this assumption in Appendix B.1. The consequences for the results are not severe, but the simple exposition would suffer.

[8] For the application of goods that are not services we can immediately interpret x^i_j as the quantity i consumes of the good bought from agent j.

[9] In our application this possibility exists in some cases. For instance, when members leave the platform, they typically have to balance a negativ time account with money.

[10] This simplifies the analysis by making demand in one market independent from constraints in other markets. We relax the assumption in Section B.2 in the Appendix.

[11] The assumption that the numeraire good is unconstrained is standard in the literature on fixed prices and belongs to the equilibrium notion. In our application, most platforms do define positive and negative limits on the holdings of time currency, but it turns out that these limits are rarely binding. An interesting restriction is to set $\underline{Z}^i_0 = \bar{Z}^i_0 = 0$ for all i, which forces each time account to be balanced. This would be a refinement of the Drèze equilibria that we study and would bring our analysis closer to that of the matching literature (e.g. Andersson et al., 2021).

(WO) exchange is "weakly orderly:" if for some commodity h, and some agents i, j, there is a trade $(\tilde{t}^i, \tilde{t}^j) \in \tau_h^i(t^i) \times \tau_h^j(t^j)$ such that $\tilde{t}^i \succ^i t^i$ and $\tilde{t}^j \succ^j t^j$, then $(\tilde{t}_h^i - \bar{Z}_h^i)(\tilde{t}_h^j - \underline{Z}_h^j) \geq 0$. In words: if there is a feasible trade that only differs from trade t on market h and on market 0 and both traders i and j would benefit from that trade, then it cannot be that the two traders are at different sides of the market in the sense of one wanting to buy less (respectively to sell more) and the other wanting to buy more (respectively to sell less).

Voluntariness (V) captures that individual agents optimize across all markets, given their constraints Z^i. $\underline{Z}^i \leq 0$ ($\bar{Z}^i \geq 0$) then ensures that i cannot be forced to buy (sell). Weak order (WO) captures that there is no pair of agents i, j who can both strictly improve by making an (additional) trade on a single market h, when the constraints on this market are relaxed. Such a trade can either be between a seller and a buyer who exchange good h for money; or between two buyers who change the amount they buy of good h without changing the total demand (for seller h). Weak order (WO) is equivalent to the following property, which is actually used in Maskin and Tirole (1984): there is no market $h (\neq 0)$ in which a Pareto improvement can be reached when ignoring the constraints on this market and keeping all other markets (except the market for the numeraire good 0) fixed.[12]

3. Results

An agent i can only afford consumption bundles x^i that are in his budget set $B^i(p) = \{x^i \in X^i | p \cdot x^i \leq p \cdot \omega^i = p_i \omega_i^i\}$. For fixed prices p, compute demand $\hat{x}^i$ of an agent i as $\hat{x}^i := \text{argmax}_{x^i \in B^i(p)} U^i(x^i)$, i.e. the consumption bundle that maximizes agent i's utility within the budget set.[13]

Definition 2 (Scarce and Non-scarce Goods)**.** Good $h \neq 0$ is called *scarce* if there is no excess supply (at fixed prices p), i.e. if $\sum_{i \in N} \hat{x}_h^i \geq \sum_{i \in N} \omega_h^i = \omega_h^h$. Otherwise, it is called *non-scarce*.

Scarce goods are in high demand, relative to their supply, while non-scarce goods are not. Note that demand and supply on each market $h \neq 0$ are independent from any other market $j \neq 0$. Desired quantity for any good $h \neq 0$, $\hat{x}_h^i$, is therefore weakly decreasing in p_h and strictly decreasing for $p_h \in (0, mu_h^i(0))$. Therefore, there is a unique price p_h^* at which the sum of desired quantities equals the endowment: $\sum_{i \in N} \hat{x}_h^i = \omega_h^h$. Considering all markets, the price vector $p^* = (1, p_1^*, \ldots, p_n^*)$ together with the market-clearing allocation x^* such that $x_h^{i,*} = \hat{x}_h^i$ constitutes a Walrasian equilibrium.[14] Hence, we define p^* as the unique market-clearing price vector that is equal to the marginal utility of all consuming agents.

The following lemma shows that scarcity of a good h can be inferred by comparing the given fixed price p_h with price p_h^*.

Lemma 1. *Good $h \neq 0$ is scarce (at fixed prices p) if and only if $p_h^* \geq p_h$.*

3.1. Characterization of fixed price equilibria

Proposition 1 (Characterization)**.** *In every FPE, each good $h \neq 0$ is allocated as follows:*

(a) *If h is non-scarce, every buyer receives the desired amount, while the seller keeps the rest, i.e. $\forall i \neq h$, $x_h^i = \hat{x}_h^i$ and $x_h^h = \omega_h^h - \sum_{i \neq h} \hat{x}_h^i (> \hat{x}_h^h)$.*
(b) *If h is scarce, every buyer receives at most his desired amount, while the seller keeps (exactly) the desired amount, i.e. $\forall i \neq h$, $x_h^i \leq \hat{x}_h^i$ and $x_h^h = \hat{x}_h^h$.*

Proposition 1 provides a clear-cut characterization of all FPE. It fully determines the allocation of all non-scarce goods and it determines the allocation of all scarce goods up to a rationing scheme. In the literature, equal rationing is sometimes imposed (e.g. in Herings and Konovalov, 2009). Our results hold for all FPE and hence for all rationing schemes. Note also that Proposition 1 holds without any assumption on $\hat{x}_h^i$ being interior. In particular, $\hat{x}_h^i \in \{0, \omega_h^h\}$ is admitted and does not change the statement. Such a clear characterization of all FPE is due to our assumptions on the utility function: Demand in one market is not affected by quantity constraints in another market. The rationing scheme for good h therefore only affects demand of good h and the numeraire. The asymmetry in the strength of the two statements (a) and (b) follows from the assumption that every agent is only endowed with one good, which means that every agent can only sell one good, while he can buy any good.[15]

If a good h is scarce but not strictly scarce, then the inequality of the second statement of Proposition 1 holds in fact with equality. Since the Walrasian prices p^* have the feature that each good h is scarce, but not strictly scarce, it follows

[12] This notion is called "weak order (O")" in Maskin and Tirole (1984). We show the equivalence of the two notions in Appendix A.1.

[13] With our assumptions on U^i, $\hat{x}_h^i$ is uniquely determined for all $i, h \neq 0$, either positive satisfying $mu_h^i(\hat{x}_h^i) = p_h$; or $\hat{x}_h^i = 0$ when $mu_h^i(0) \leq p_h$.

[14] When there are corner solutions, other Walrasian equilibria that differ in equilibrium prices may exist.

[15] Relaxing this assumption, would lead to results for non-scarce goods that are fully analogous to the results with scarce goods (see Appendix B.1). Such results are slightly weaker since the allocation of non-scarce goods then also depends on the rationing scheme. However, loosening this assumption would not undermine the substance of the results.

that in Walrasian equilibrium, which is a special case of a FPE, no agent is constrained, while markets clear. However, for generic prices $p_h \neq p_h^*$ at least one agent is constrained from buying the desired amount of a scarce good h and the seller of a non-scarce good h is constrained from selling the desired amount in the sense that he consumes more of his own good than he would buy at these prices.

3.2. Inefficiency of fixed price equilibria

We now turn to efficiency.

Definition 3 (Pareto Efficiency and Constrained Efficiency)**.** An allocation x is *Pareto efficient (PE)* if $\nexists t \in T$ such that $x + t$ Pareto dominates x. An allocation x is *constrained efficient (cPE)* if $\nexists t \in \tilde{T}$ such that $x + t$ Pareto dominates x.

The notion of Pareto efficiency is stronger than the notion of constrained efficiency because it admits more general improvements. For Pareto efficiency we consider any other trade that is mutually compatible, while constrained efficiency only considers mutually compatible trades that obey the budget feasibility for the fixed prices p. Instead of requiring that every agent wants to consume a strictly positive amount of every good, i.e. interiority, we make a much weaker assumption on the attractiveness of different goods.

Definition 4 (Weak Interiority)**.** An economy satisfies *weak interiority* if the following holds for every market $h \neq 0$.

(i) If h is non-scarce, then there is another non-scarce good $k \neq h$ such that $\hat{x}_k^h > 0$, i.e. the seller of a non-scarce good h demands at least one other non-scarce good.
(ii) If h is scarce, then $\hat{x}_h^h > 0$, i.e. the seller of a scarce good h demands a positive amount of it.

With these notions in hand, we can formalize the inefficiency, not only with respect to Pareto efficiency, but also with respect to constrained efficiency.

Proposition 2 (Inefficiency)**.** *Suppose a non-scarce good h and at least one agent $i \neq h$ exist such that $\hat{x}_h^i > 0$. Then no FPE is Pareto efficient. Suppose $p_h^* \neq p_h$, $\forall h$, and weak interiority is satisfied. Then no FPE is constrained efficient.*

The first statement of Proposition 2 is a standard inefficiency result. In the proof for the second part, we show that under the condition of weak interiority, there is a chain of agents such that each pair in the chain can strictly improve by bilateral trade on a single market.[16] Herings and Konovalov (2009) derive a similar inefficiency result. For a specific rationing scheme (equal rationing) and interiority they show that a necessary condition for a FPE to be constrained efficient is that every constrained agent must be constrained in each constrained market. Our result is stronger in the sense that we show that under *weak* interiority no rationing scheme *exists* such that a FPE is constrained efficient. This is possible due to our assumption of quasi-linear preferences.

The inherent type of inefficiency emerging from the combination of fixed prices and decentralized trade is easiest to see by assuming prices fixed to $p \equiv (1, \ldots, 1)$ and interiority. By Proposition 1 a supplier i of a non-scarce good derives then a marginal utility of 1 from each non-scarce good $h \neq i$. However, his marginal utility from good i is strictly smaller. Therefore, any two suppliers of a non-scarce good could improve by exchanging some amount of their goods directly, without using the numeraire good in the transactions. This will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In some sense the prices of the two goods are "too high." A similar issue occurs for scarce goods: prices are "too low" such that despite the high demand, a supplier of the scarce good is not willing to offer a sufficient amount of it, while she would do so in exchange for another good that she values highly. This shows how decentralized trade fails to enable even simple Pareto improving trades when prices are fixed.

3.3. Fixed Price vs. Walrasian equilibrium

We now compare the Walrasian equilibrium and FPE, first with respect to the amount traded and then with respect to incomes.[17]

Proposition 3 (Less Trade)**.** *In every FPE t, the total amount traded of any good $h \neq 0$ is smaller than in the Walrasian equilibrium t^*, i.e. $\sum_{i \neq h} t_h^i \leq \sum_{i \neq h} t_h^{i,*}$. For non-scarce goods h, every single buyer $i \neq h$ buys less than in the Walrasian equilibrium, i.e. $t_h^i \leq t_h^{i,*}$, $\forall i \neq h$.*

The result follows from Proposition 1 and the law of demand. Suppose that the fixed price p_h of a good h does not coincide with the Walrasian price p_h^*. If h is non-scarce, $p_h^* < p_h$ (Lemma 1). Since buyers of non-scarce goods are not constrained (neither in the FPE nor in the Walrasian equilibrium), they would buy more in the Walrasian equilibrium. If good

[16] In the terminology of Herings and Konovalov (2009), this means that no fixed price equilibrium is "B-p efficient," which is an even weaker notion of efficiency than constrained efficiency.

[17] It is important to note that this is not automatically a comparison between fixed and flexible prices. Price flexibility may lead to Walrasian prices under a price-taking assumption, but it may also lead to monopoly prices. FPE on the other hand, nest both Walrasian and monopoly prices as special cases, as we further discuss in Section 4.

h is scarce, $p_h^* > p_h$ (Lemma 1). Since sellers of scarce goods are not constrained (neither in the FPE nor in the Walrasian equilibrium), they would sell more in the Walrasian equilibrium.

To interpret Proposition 3, notice that voluntary trade is not generally maximized under the Walrasian price. First, relaxing the assumption of a uniform price for each good, i.e. admitting price discrimination, can foster more trade.[18] Second, even with uniform pricing, there are "counter-examples" in which there is more trade in a FPE than in a Walrasian equilibrium. These "counter-examples" however do not satisfy our assumption of additive separable preferences.[19] Another result to put Proposition 3 in perspective concerns the matching algorithm for time exchange proposed by Andersson et al. (2021): It maximizes trade under fixed prices with the additional constraint that every time account must be balanced. If we apply this additional constraint to our fixed price equilibria, we actually refine the equilibrium notion by restricting attention to those FPE that satisfy time-balance. Hence, the implication of Proposition 3 for this case is that even weakly less is traded.

The result on less trade also has implications for the incomes of the market participants. Let $y = (y^1, \dots, y^n)$ denote the income distribution with $y^h = \sum_{i \neq h} t_h^i \cdot \ p_h$ being the income of the supplier of good h. Since suppliers of scarce goods sell less with fixed prices (by Proposition 3) and fixed prices are lower than their Walrasian prices in equilibrium (by Lemma 1), their income is lower under fixed prices. Suppliers of non-scarce goods also sell less in the FPE, but fixed prices for their goods are higher than Walrasian prices. Whether the overall effect on income is positive or negative depends on the price elasticity of demand. The relevant prices are $p = (1, p_1, \dots, p_n)$ and $p^* = (1, p_1^*, \dots, p_n^*)$ and the corresponding demand is $Q_h := \sum_{i \neq h} \hat{x}_h^i$ and $Q_h^* := \sum_{i \neq j} x_h^{i,*}$. Hence, we define the (discrete) price elasticity of demand as $\varepsilon_h := \frac{\Delta Q_h}{\Delta p_h} \cdot \frac{p_h}{Q_h} = \frac{Q_h^* - Q_h}{p_h^* - p_h} \cdot \frac{p_h}{Q_h}$.

Corollary 1 (Income)**.** *Suppose that every good $h \neq 0$ faces positive demand for the fixed price p, i.e. $Q_h > 0$. Then moving from any fixed price equilibrium to the Walrasian equilibrium affects the incomes as follows:*

(i) *Suppliers of a scarce good receive a weakly higher income.*
(ii) *Suppliers of a non-scarce good receive a weakly higher income if and only if their good's demand is sufficiently elastic, i.e. $|\varepsilon_h| \geq \frac{p_h}{p_h^*} (> 1)$.*

Suppliers of strictly scarce goods benefit from the introduction of Walrasian prices by receiving an income boost, while suppliers of non-scarce goods receive lower incomes when demand is inelastic.

These unequal effects on the incomes of the two types of suppliers may well, but need not, lead to more pronounced income inequality under flexible prices than under fixed prices.[20]

4. The platform's problem

We address the platform's optimization problem in two complementary subsections. First we focus on the optimal price regime in a setting where all agents choose to participate. Then we investigate who participates as a function of the membership fee.

4.1. The platform's choice of price regime

We consider a model variation with the following timing.

I The platform chooses membership fee $f > 0$ and whether prices are fixed or flexible. If prices are fixed, the platform also determines a price $p_i > 0$ for each good $i \neq 0$.
II Every agent decides whether to become a member of the platform or not. If prices are flexible, every member $i \in M (\subseteq N)$ decides on the price of her product $p_i > 0$.
III Payoffs are received. Let $x(p, M)$ be the FPE allocation for price $p = (1, p_1, \dots, p_{|M|})$ and equal rationing when the set of members is $M \subseteq N$. The payoff to agent i is

$$\pi^i(p, M) = \begin{cases} U^i(x^i(p, M)) - f, \text{ when } i \in M \\ U^i(\omega^i), \text{ otherwise} \end{cases} \quad (1)$$

The payoff for the platform is $\Pi(p, M) = f * |M|$.

Payoffs, which are obtained in Stage III, reflect the outcome of members M who engage in trade on a platform with prices p.[21] Observe that fixed price equilibria nest the outcomes under flexible prices as the FPE for a previously set price.

[18] When a homogeneous good can be traded at different prices, trades below and above the Walrasian equilibrium price take place when a seller and a buyer with a corresponding willingness to pay, respectively to sell, meet. With this insight Bergstrom (2004) theoretically underpins what is referred to as the first classroom market experiments: Chamberlin (1948) finds *more* trade among his student participants than predicted by Walrasian equilibrium.

[19] In Appendix B.2 we discuss why the less trade result does not generalize to preferences that are not additive separable, but how it generalizes partially to preferences that are not quasi-linear.

[20] Comparisons of income distributions have to be distinguished from welfare comparisons. Whether an agent is better off in the Walrasian equilibrium or in the FPE depends not only on her income, but also on the prices of the goods she demands, and on the quantity constraints she faces at the scarce goods.

[21] A member i receives a payoff equal to the utility in the FPE among members M with prices p and equal rationing net the fee. Equal rationing only applies when there are strictly scarce goods, as for non-scarce goods and weakly scarce goods, the equilibrium allocation is uniquely determined by Proposition 1. A non-member i receives a payoff equal to her endowment: $U^i(\omega^i)$.

For the platform's payoff, we consider the generated revenue. Typically for online platforms variable costs are very small. Then maximizing profit coincides with maximizing revenue.

To simplify the exposition we assume that all agents are homogeneous, i.e. $\omega_i^i = w > 0$ for all $i \neq 0$ and $\omega_i^j = 0$ for $i \neq j$; as well as for each agent i, we have

$$U^i(x^i) = x_0^i + u(x_1^i) + \ldots + u(x_i^i) + \ldots + u(x_n^i),$$

where the function $u := u_h^i$ for all $i, h \neq 0$ satisfies all properties assumed in Section 2. In Appendix A.8, we formally solve this game for subgame perfect Nash equilibria that satisfy the following selection criteria: (i) The platform treats all agents equally, (ii) agents' strategies are symmetric, and (iii) agents coordinate on participation.

In Stage III, Proposition 1 together with equal rationing characterizes the payoff for every price vector p. In the subgames of Stage II, the selection criteria reduce the set of equilibria to some where all agents participate and others where none participates. If the platform chooses a too high fee f in Stage I, agents will not participate and hence the platform's revenues are zero. For lower fees f and with flexible prices, it turns out that every agent i participates and chooses a price $p_i^M > p_i^*$. The highest revenue with flexible prices is hence $\bar{f} * n$ obtained for the fee $\bar{f} = U^i(x^i(p^M, N)) - U^i(\omega^i)$, with $p^M = (1, p_1^M, \ldots, p_n^M)$. Importantly, when choosing fixed prices in Stage I, the platform can replicate flexible prices by setting prices $p = p^M$. However, a platform can do better by choosing lower prices and higher fees. In particular, the gross payoff $U^i(x^i((1, \rho, \ldots, \rho), N))$ is maximized for the market clearing prices $\rho = p_i^*$, as the following Lemma shows.

Lemma 2. *When agents are homogeneous, the gross payoff $U^i(x^i((1, \rho, \ldots, \rho), N))$ of every agent i is a single-peaked function in the common price ρ, where the peak is at the Walrasian price $\rho = p_i^*$.*

As a consequence, we get the following solution to the game.

Observation 1. In the subgame perfect Nash equilibrium (selected by our criteria), the platform chooses fixed prices equal to Walrasian prices $p = p^*$ and fee $f = nu(\frac{w}{n}) - u(w)$.

It is intuitive that a perfectly informed platform can always set a price that is at least as good (for any goal), than prices set by the members. Here the fixed price is even strictly better, as flexible prices would lead to monopoly pricing $p_i^M > p_i^*$. While inefficiency of monopoly pricing is well-known, we find constrained inefficiency by Proposition 2, as for monopoly prices all goods i are actually non-scarce. Note that the choice of the platform not only maximizes its profit, but it also maximizes welfare (by Lemma 2) as well as the amount of trade (by Proposition 3).

An interesting trade-off emerges when the platform cannot perfectly determine p^*, e.g. due to uncertainty about agents' preferences. Then there is a trade-off between potentially too high or too low fixed prices and a too high monopoly price. Lemma 2 has some implications for this case. For any two prices which are lower (or higher) than p^*, payoffs are always (weakly) higher under the price that is closer to p^*. Whenever the platform observes the monopoly price, it can therefore increase payoffs by setting a slightly lower price. In case it can neither observe the market clearing nor the monopoly price, flexible prices might be better in approximating the (efficient) Walrasian price.

In the special case of homogeneous agents that we have analyzed, it is natural that there is a common price. When agents are heterogeneous, but the platform is not perfectly informed about the Walrasian prices (e.g. only knows their distribution) then it might also choose a common price. Hence, there is a similar trade-off between inefficiency due to market power with flexible prices and inefficiency due uniformly fixed prices. The more heterogeneous the agents are and the lower their market power (p_i^M close to p_i^*), the more favorable seem flexible prices.

4.2. The platform's choice of membership fee

We have focused in the last subsection on the optimal choice of price regime in a setting where all agents choose to participate. Now, we focus on which set of agents participate, given a price vector p. In this variation of the model the timing is as follows.

I The platform chooses membership fee $f > 0$.
II Every agent decides whether to become a member of the platform or not.
III Payoffs are received. An agent i receives payoff $\pi^i(p, M)$, as defined in Eq. (1), where again $M \subseteq N$ is the set of members. The payoff for the platform is $\Pi(p, M) = f * |M|$.

Payoffs, obtained in Stage III, again reflect the outcome of members M who engage in trade on a fixed price platform with given prices p. Stage II consists of subgames that start at a fee f determined in Stage I. An equilibrium in the subgame satisfies that no agent can benefit by unilaterally changing membership status. This means for a member $i \in M$ that

$$WTP^i(p, M) := U^i(x^i(p, M)) - U^i(\omega^i) \geq f,$$

i.e. having a willingness to pay for the platform that weakly exceeds the membership fee. Similarly, this means for a non-member $i \notin M$ that $WTP^i(p, M \cup \{i\}) \leq f$. Stage II is prone to induce a multiplicity of equilibria. In any subgame with multiple

Table 1
Example of equilibrium sets of members in Stage II for platform with prices $p = (1, 1, \ldots, 1)$. Left part of table shows willingness to pay $WTP^i(p, M)$ for members and "out" for non-members. Right part shows fees at which these sets of members are equilibria and the maximal revenue.

	Agent 1 WTP^1	Agent 2 WTP^2	Agent 3 WTP^3	Agent 4 WTP^4	Agent 5 WTP^5	equilibrium if	revenue $\bar{f} * \lvert M\rvert$
NE 1	0.159	0.271	0.238	0.208	0.084	$f \le 0.084$	0.419
NE 2	0.130	0.241	0.221	0.188	out	$f \in [0.084, 0.130]$	0.518
NE 3	out	0.202	0.200	0.184	out	$f \in [0.130, 0.184]$	0.552
NE 4	out	out	out	out	out	$f \ge 0$	0

equilibria, we follow the convention to select those with the highest number of members. For the platform, this selection is preferred, as it maximizes revenue. Anticipating Stages II and III, the platform chooses fee f in Stage I.[22]

To investigate the solution of this problem, let us use a functional form that is simple enough to be tractable, but rich enough to capture heterogeneity of agents. Suppose for every agent $i \in M$, utility of consuming any $x^i \in X^i (= \mathbb{R} \times \mathbb{R}_+^{|M|})$ is

$$U^i(x^i) = x_0^i + \alpha_1 \log(1 + x_1^i) + \alpha_2 \log(1 + x_2^i) + \ldots + \alpha_{|M|} \log(1 + x_{|M|}^i), \tag{2}$$

with $\alpha_h \in [1, 2]$ for any good $h \neq 0$. Hence, the higher the parameter α_h, the more attractive good h is for all agents. Moreover, let us set $\omega_i^i = 1$ and $\omega_j^i = 0$ for all $i, j \in N$ with $i \neq j$, i.e., every agent i is endowed with one unit of her good. We start with an example.

Example 1 (Equilibrium Sets of Members)**.** Consider agents $N = \{1, 2, 3, 4, 5\}$ whose goods' attractiveness are characterized by the following parameters: $\alpha_1 = 1.03, \alpha_2 = 1.2, \alpha_3 = 1.25, \alpha_4 = 1.3, \alpha_5 = 1.9$. Let prices be fixed at $p = (1, \ldots, 1)$, where the price vector has length $|M|$.

It turns out that there are four different equilibrium sets of members in Stage II, as summarized in Table 1. First, $M = N$ is a Nash equilibrium in subgames where the fee f is smaller than 0.084, which equals the willingness to pay of Agent 5. Hence, with this set of members the highest revenue the platform can achieve is $\bar{f} * |M| = 0.084 * 5 = 0.419$. A higher revenue can be achieved in equilibrium 3, where Agents 1 and 5 are not members. We have this equilibrium in subgames with $f \in [0.130, 0.184]$ such that the highest revenue the platform can achieve is $\bar{f} * |M| = 0.184 * 3 = 0.552$.

In Example 1 the platform excludes those agents with extreme attractiveness, extremely low $\alpha_1 = 1.03$ and extremely high $\alpha_5 = 1.9$, as they have the lowest willingness to pay.

Let us now investigate a bit more generally which agents select into fixed price platforms by analyzing the willingness to pay $WTP^i(p, N)$. We approximate the distribution of the willingness to pay $WTP^i(p, N)$ among a set of agents N with given attractiveness $(\alpha_1, \ldots, \alpha_n)$ by the change in willingness to pay of a single agent $\frac{\partial WTP^i}{\partial \alpha_i}$.[23] Again, we use homogeneous logarithmic utility, as defined in Eq. (2), set all endowments to $\omega_i^i = 1$ for $i \neq 0$, and fix all prices at $p_i = 1$. First we observe that a good i is scarce if and only if $\alpha_i \ge 1 + \frac{1}{n}$.[24] Now, we can apply Proposition 1 to determine an agent i's willingness to pay as a function of the parameter α_i.

Observation 2. Any agent i's willingness to pay $WTP^i((1, \ldots, 1), N)$ is single-peaked in the own good's attractiveness α_i with peak slightly below the threshold $1 + \frac{1}{n}$, which distinguishes scarce goods from non-scarce goods.

This observation clearly suggests that fixed price platforms exclude rather those agents with very unattractive (non-scarce) goods, as well as those with very attractive (scarce) goods.

As a comparison, let us now look at the willingness to pay under flexible prices, which are either interpreted as Walrasian prices p^* (price-taking assumption) or monopoly prices p^M (price-making assumption).

Observation 3. Any agent i's willingness to pay under flexible prices, $WTP^i(p^M, N)$ or $WTP^i(p^*, N)$, is increasing in the own good's attractiveness α_i.

It is intuitive that the platform is more valuable for agents with highly attractive goods, as both prices p_i^M and p_i^* of an agent i are increasing in α_i. In sum, fixed prices lead to a starkly different conclusion about who selects into a platform if there is any selection. While for flexible prices (be it Walrasian or monopoly prices) those with the most attractive goods select into the platform, under fixed prices these agents are typically excluded.[25]

[22] Formally, for any subgame starting at fee f, let $\eta(f)$ denote the number of members in those Nash equilibria with the highest number of members. Then, the platform faces the following maximization problem: $\max_{f>0} f * \eta(f)$.

[23] This approximation abstracts from the following effect: If $WTP^i(p, N)$ is decreasing in α_i, there might still exist two agents j and k (with $\alpha_j, \alpha_k \approx 2$) such that $\alpha_j < \alpha_k$ and $WTP^j(p, N) < WTP^k(p, N)$. The reason is that k's offer is less relevant than j's because k is not offering much (close to zero), while j offers more. For large n, this effect vanishes.

[24] As shown in the proof of Observation 2.

[25] We have assumed homogeneity of utility functions which excludes differences in the willingness to pay due to being a consumer. For heterogeneous consumers, obviously those with the highest valuation would rather select into the platform, independent of the pricing regime.

Table 2
Description of different platforms with time-based currencies. Years is the recorded time span. Members are all participants of a platform who had at least one transaction with another member. Transactions (cum. transactions) is the number of (cumulative) transactions on the platform. Trade volume is the money in the platform-specific currency spent on trades. The Gini coefficient measures inequality in terms of sales volume (income). To make the platforms comparable, the number of transactions, the trade volume and the Gini coefficient are normalized by taking the average amount per member per year (last three columns).

ID	years	members	cum. transactions	av. transactions	av. trade volume	av. Gini
F1	5.6	215	1559	4.55	10.51	63.1
F2	7.9	330	5094	6.21	13.34	63.5
F3	8.7	324	4175	5.20	14.45	64.5
F4	9.6	708	12,513	6.70	11.92	64.6
W1	5.6	179	2804	8.45	22.43	67.2
W2	6.7	118	2975	13.79	13.60	60.1
W3	10.4	1037	69,346	16.64	50.52	74.1

5. An empirical illustration

5.1. Transaction patterns

Our theoretical investigation points to the difficulties of decentralized trade under fixed prices and to differences when compared with flexible prices. The model applies in particular to time exchange markets. These are the purest real-world examples of exchange economies we can think of. Concretely, these are marketplaces for service exchange, which facilitate decentral trade through a platform-specific currency that is related to time. Often, but not always, all prices are fixed and equal, e.g. any hour of service yields one hour on the time account for the supplier and costs one hour for the consumer. Such markets have existed at least since the nineteenth century (see e.g. Warren, 1852), but it was much more recently that many such markets have been created all around the world.[26]

We now set out to describe real transaction patterns of several such platforms to explore how decentralized trade under fixed prices works compared to more flexible prices. For seven platforms, we obtained data of all transactions made between 2008 and 2016.[27] Each platform has a set of rules on how to trade on them. These rules are highly similar to each other on all platforms with one crucial difference: Prices are fixed to a higher or lower degree. Four platforms strictly fix prices by tying them to hours of service, as the currency is purely time-based. The three others only provide recommendations for a possible conversion between hours of service and price, giving more leeway to the sellers to define the price in the artificial currency themselves.[28] To be clear, we do not make any causal claim, but only describe how fixed prices are associated with trading patterns.[29]

Table C.1 provides summary statistics about the platforms. According to the rules on how to set prices, we organize the platforms into four with fixed prices labeled F1,...,F4 and three with rather flexible prices labeled W1,...,W3. Within both categories the platforms are ordered and labeled according to the length of our recordings (see column *years*). Members are defined as participants who engaged in at least one transaction with another participant. In total, we have data on 2911 members and of 98,527 transactions (not counting system transactions such as the payment of an annual membership fee). All these platforms charge a membership fee, none charges a transaction fee.[30]

The focus of Table C.1 is on the last three columns, which are normalized to make the platforms comparable and which speak to our theoretical results. We have in mind that fixed price platforms have set all prices to be all equal such that most likely some goods are scarce and some are non-scarce, while platforms with more flexible prices could be assumed to come closer to the Walrasian ideal. The fact that under fixed prices some welfare improving trades are not realized (Proposition 2) then suggests that we should observe less transactions under fixed prices. Indeed, Table 2 shows that *the number of transactions (per member per year) is substantially smaller for the platforms with fixed prices (F1-F4).* On average the platforms with fixed prices only have 6.0 transactions per member and year, while those with flexible prices have 16.9.[31] The fact that fixed prices induce less trade than Walrasian prices in terms of quantities (Proposition 3) cannot be directly assessed in this data set since quantities are missing for many transactions. Instead, we have the number of transactions and the trade volume, which is the traded quantities weighted by the prices, as two proxies. Both proxies are higher on

[26] For instance, already in 2011, 300 registered "time banks" have been counted only in the US, which is just one of 34 countries with such institutions (Cahn, 2011). There is a broad range of services offered, from ironing clothes, mowing someone's lawn to looking after children, or teaching a certain craft. The creation of such systems does not imply that they are successful or popular.

[27] More details about the data set are provided in Appendix C.1.

[28] See Appendix C.2 for the corresponding pricing rules.

[29] There could be unobserved variables, e.g. the heterogeneity between goods, that affect both the trading patterns and the platform's choice on how to fix prices. Moreover, trading patterns of a given platform can be influenced by other factors, e.g. characteristics of the region or marketing activities of the platform operator.

[30] While some membership fees involve not only the platform-specific currency but also a part that is paid in a standard currency, there are no starter packages (or "loot boxes") that allow to buy endowment, including the currency, for money.

[31] The averages and standard deviations are displayed in Appendix C.3.

Table 3
Network statistics. Nodes are the members of a platform. Arcs are the customer-supplier relationships. Density is the number of present arcs over all potential arcs. Centralization measures inequality with respect to the number of customers (indegree). Transitivity is the fraction of transitive triples, i.e. how often a customer's customer is an own customer. Clustering is the average clustering coefficient of the undirected network, i.e. given an average member, how often are two of its trade partners also trade partners themselves.

id	nodes	arcs	arcs/node	density	centralization	transitivity	clustering
F1	215	695	3.2	0.015	0.11	0.33	0.10
F2	330	1593	4.8	0.015	0.13	0.20	0.08
F3	324	1756	5.4	0.017	0.10	0.17	0.08
F4	708	4828	6.8	0.010	0.14	0.24	0.17
W1	179	1197	6.7	0.038	0.27	0.46	0.21
W2	118	1150	9.7	0.083	0.48	0.53	0.41
W3	1037	23,071	22.2	0.021	0.41	0.40	0.30

average for the flexible prices (W1-W3), but in contrast to the number of transactions, the trade volume is not consistently higher on the platforms with flexible prices. Finally, the fact that Walrasian prices induce an income boost of some agents (Corollary 1) suggests that equality of the income distribution is affected. Therefore, we assess inequality of incomes. As the Gini coefficients in the last column show, inequality, is particularly high on two platforms with flexible prices W1 and W3. A closer inspection of the income distributions shows that these differences are driven by the relatively high number of transactions of top earners on these platforms.[32]

5.2. Trade networks

We analyze the trade networks that are implied by the transactions on each platform. Each member is a node in a network and directed links, arcs, represent the customer-supplier relationships.[33]

Table 3 reports several network statistics for each platform. The platforms are ordered as before. The number of arcs per node is the number of customers an average member of the platform has (which coincides with the number of suppliers an average member has). The density is the fraction of present arcs over all possible arcs. The table reports that *the trade networks are denser for the platforms with more flexible prices (W1-W3).* On average the platforms with fixed prices only have 1.4% of all possible customer-supplier relationships established, while those with rather flexible prices have a density of more than 4.7%. This observation might again reflect our theoretical prediction of inefficiency (Proposition 2), which was based on the argument that mutually beneficial trades are not realized when prices are fixed; and our prediction of less trade (Proposition 3), now with respect to the number of customer-supplier relations.

Concerning the distribution of customers, indegree centralization measures inequality with respect to the number of customers (Freeman, 1978). The measure ranges from zero, attained when every member has the same number of customers, to a one, attained in a star network, in which one member has all customers and no other member has any customer. Table 3 reports that *the trade networks under rather flexible prices (W1-W3) are more centralized and hence more unequal*. This observation indicates that under flexible prices some agents become successful suppliers in the sense of serving substantially more customers than the other members.

Finally, we look at two measures of clustering (or network "closure"), which, roughly speaking, answers how often two trade partners of a given member are trade partner themselves. Transitivity is the usual notion for binary relations and, as all measures before, it uses the directed network of buyer-supplier relations. In contrast, average clustering is more common in network science and it is based on the undirected network, which considers any pair of customer and supplier as linked trade partners, independent of the direction of their relation.[34] Table 3 reports that *both transitivity and average clustering are higher for the trade networks with rather flexible prices (W1-W3).*

This means that under flexible prices it happens more often that a customer's customer is an own customer and that two trade partners of a member are trade partners themselves. The latter structure is often argued to be important to foster cooperative behavior and trust (Coleman, 1988; Buskens, 2002), because it increases the potential of sanctions, e.g. for not delivering the promised service in the promised quality at promised time.

These observations give an indication that under fixed prices different trade networks may emerge than under flexible prices. In particular, they suggest that more flexible prices are associated with higher density, higher centralization, and higher clustering,[35] in addition to the higher number of transactions that we observed before.

[32] See the Lorenz curves in the Appendix C.4.

[33] Two of these networks are illustrated in Fig. C.3 in Appendix C.5. The networks that we study differ from typical buyer-seller networks (e.g. Kranton and Minehart, 2001; Corominas-Bosch, 2004) since the market participants that we study act as both buyers and sellers.

[34] A more extensive explanation of these two measures is provided in Appendix C.5.

[35] These measures are not independent of each other.

6. Discussion

Given the theoretical findings and empirical observations above, why do platforms for peer-to-peer exchange often restrict price setting and what are the advantages and disadvantages of these markets in comparison to other market forms?

First, it must be clear what fixed and what flexible prices really mean. A well-informed platform can reach any goal (profit, welfare, equality, ...) at least as good by fixing the right price as with leaving them flexible (Section 4.1). However, realistically, the platform lacks information to set prices perfectly, and hence fixes prices heuristically, e.g. making them all equal. As a consequence some goods are scarce and some are non-scarce which induces inefficiency, as we have elaborated in Section 3. Flexible prices on the other hand need not lead to Walrasian prices and induce efficiency, but can also mean that monopoly prices are chosen by the sellers. This is another kind of inefficiency in which all prices are higher than optimal. In fact, the monopolistic prices are nested as a special case of a FPE where all goods are formally non-scarce and hence the allocation is also constrained inefficient by Proposition 2. A benevolent platform has to trade-off these two kinds of inefficiency. The trade-off of a profit-maximizing platform is similar, as high surplus of the platform members can be absorbed by the platform, e.g. with membership fees. It seems that insufficient information of the platform and heterogeneity of goods on the platform rather speaks for flexible prices, while fixed prices are most reasonable, when the goods are of similar quality and the platform operator is well-informed.

Second, besides efficiency, simplicity is also a desirable feature of a well-designed market. Market participants should be able to understand the conditions of their transactions without much effort. If there are high transaction costs for finding mutual agreements on how much to pay for certain services, it can be reasonable to rely on focal prices, which are suggested by a platform operator.[36] A third reason for fixing prices is price stability. The value of a platform-specific currency in real terms, i.e. in terms of future consumption possibilities, might otherwise fluctuate and would therefore be hard to assess. By fixing prices, a platform operator can induce price stability and potentially reduce a market participant's uncertainty about consumption possibilities in the future.

A fourth – and in the application of time exchange markets possibly the most important – motivation for fixed prices are social preferences. Certain prices could be perceived as *fair* such that (a) *procedural fairness* is a motive to engage in these transactions;[37] or it could be that the resulting allocation is considered more fair, than the equilibrium allocation under flexible prices such that (b) *distributional fairness* is the motive. If the former motive, (a) procedural fairness, is predominant, the question arises whether there are Pareto superior allocations given the restriction that services are only exchanged according to the fixed prices. Our paper provides an answer to this question by showing that the FPE allocations are constrained inefficient and that Pareto improvements often only necessitate simple trades. Participants motivated by procedural fairness could hence agree to a different allocation mechanism that keeps the same prices, but leads to Pareto superior outcomes, e.g. a centralized matching procedure. The matching algorithm that is proposed for time exchange by Andersson et al. (2021) maximizes trade and is hence efficient. While keeping procedural fairness with respect to fixed prices, it introduces another procedural fairness issue since the first agent in the order receives her preferred bundle, which is not true for the last agent.[38]

Concerning (b) distributional fairness, our paper shows that price restrictions do affect the distribution of incomes. If the income distribution of the fixed price equilibrium is considered as more desirable and this motive is predominant, the question arises whether there are alternative (market) mechanisms that lead to Pareto superior outcomes, given the agents' social preferences. For instance, more trade without much higher inequality could be induced by a competitive market combined with some redistribution of income that is accepted as fair (e.g. Alesina and Angeletos, 2005; Almås et al., 2010; Bénabou and Tirole, 2006). Extending the view on distributional fairness to those agents who do not join the platform, our observation that flexible prices favor the strongest sellers and rather exclude the weakest suggests another reason for platforms with fixed prices. Such platforms could be created with the intention to integrate agents who would otherwise be left out.[39] When studying the willingness to pay to participate in a platform, we find that fixed price platforms indeed do not favor the agents with the most attractive goods. However, also those with the least attractive goods might well be excluded since the fixed price is too high for their goods such that they cannot benefit much from trade.

Since the inefficiency we discover is related to a lack of trade, merely fostering trade would likely increase welfare. For instance, when there are many goods on the market for which prices are too high, the lack of demand for these non-scarce goods is a key driver of a low trade volume. The platform operator could mitigate this problem by reducing annual membership fees for those who buy frequently (i.e. a negative transaction fee for buyers). This is in the logic of Rochet and Tirole (2003), who show how subsidizing one side of the market can increase welfare. Merely increasing the amount of

[36] When Einav et al. (2016) study more general peer-to-peer markets – not requiring that every buyer is a seller and vice versa – they discuss examples of flexible prices, which are more or less simple. For instance, they argue that fixing an hourly wage for a service is more appropriate than finding lump sum prices for certain services.

[37] In fact, the origin of time-dependent currencies is the postulate that every hour of work should have the same value (Warren, 1852).

[38] A crucial property for centralized mechanisms is strategy-proofness. Andersson et al. (2021) show that their mechanism is indeed strategy-proof on a domain of preferences that fits this application reasonably well. For general preferences, however, such a mechanism that is strategy-proof, individually rational and efficient is impossible (Sönmez, 1999).

[39] Indeed, there is some evidence that time exchange markets attract people who do not find access in other markets, such as the local labor market (Seyfang, 2003).

platform-specific currency, e.g. by offering the platform-specific currency for money, does not have any effect in our model (due to our assumption of quasi-linearity), but might still help in reality (see e.g. Sweeney and Sweeney, 1977).

A more radical change of the platform would be to fully abandon prices and the platform-specific currency with it. Reputation mechanism might then enable fruitful gift exchange.[40] Online platforms have powerful tools to manage reputation when they work as "infomediaries" (e.g. Belleflamme and Peitz, 2015, Chapter 23). Buyers could rate and review sellers such that instead of receiving a precise amount of a certain currency, a member who delivers a good or service increases his reputation. This infomediary role of the platform would also help to reduce uncertainty about the quality of the goods and services, which is probably an additional important impediment for trade.

7. Conclusion

We have analyzed platforms for peer-to-peer exchange (where every seller must also be a buyer and vice versa). These are closed exchange economies, on which price setting is often restricted and markets therefore do not clear. Assuming quasi-linear preferences allowed us to characterize the set of fixed price equilibria. Allocations are typically constrained inefficient, i.e. there are Pareto improvements even within the given price system. Moreover, we can show that the amount of trade under fixed prices is always lower than under competitive prices. One implication is that flexible prices mean an income boost for some suppliers, but not for others. Interestingly, agents with highly attractive goods, e.g. highly productive workers, have only weak incentives to participate in such platforms. Complementing these theoretical findings with an empirical illustration of several real platforms with time-based currencies then feeds our discussion of advantages and disadvantages of peer-to-peer platforms with price restrictions.

Our methodological approach is innovative in that it combines traditional economic theory with a current online phenomenon and also makes use of techniques from network analysis. The main results show that fixed prices come at a high cost (since they lead to a constrained inefficient outcome and to less trade than competitive prices). This finding relates back to known inefficiency results (Younés, 1975; Maskin and Tirole, 1984; Herings and Konovalov, 2009), which seem to become vital and tangible in our setting. By analyzing and illustrating how platforms for peer-to-peer exchange are affected by fixed prices, we hope to provide lessons that are not restricted to these markets, but can be addressed in many markets with price restrictions.

Declaration of Competing Interest

None.

Appendix A. Proofs

A1. Lemma A.1

Lemma A.1 (Weak Order)**.** *Property weak order (WO) as defined in Definition 1 is equivalent to the following property of Maskin and Tirole (1984):*

(O") *exchange is weakly orderly: for all markets h, there exists no alternative feasible vector* $\tilde{t} \in \prod_i \tau^i_h(t^i)$ *such that, for each i,* $\tilde{t}^i \succsim^i t^i$ *with at least one strict preference.*

Proof. Clearly, (O") implies (WO) because if (WO) is violated, then there exists a pair i, j and a trade $(\tilde{t}^i, \tilde{t}^j)$ which is a Pareto improvement. On the other hand suppose (WO) is satisfied. Then there is no such pair as shown below.

Suppose there is a Pareto improvement $\tilde{t}$ concerning market h. Then at least one agent i must be better off: $\tilde{t}^i \succ^i t^i$. Hence, $\tilde{t}^i_h \neq t^i_h$. Assume first that $\tilde{t}^i_h > t^i_h$ (i.e. i would like to buy more of h or sell less of it). By $\sum_i \tilde{t}^i_h = 0$ there must be some $j \neq i$ with $\tilde{t}^j_h < t^j_h$, i.e. who sells more or buys less of h. Since $\tilde{t}$ is a Pareto improvement, $\tilde{t}^j \succsim^j t^j$. Thus, either $\tilde{t}^j \succ^j t^j$ and we are done or $\tilde{t}^j \sim^j t^j$. In the latter case, consider $\hat{t} := \frac{t+\tilde{t}}{2}$. Strict convexity implies that $\hat{t}^j \succ^j t^j$. Moreover, $\hat{t}^i \succ^i t^i$. Now, analogously for $\tilde{t}^i_h < t^i_h$ there is a j with $\tilde{t}^j_h > t^j_h$ and $\tilde{t}^j \succsim^j t^j$. Again, we have either $\tilde{t}^j \succ^j t^j$ or $\hat{t} := \frac{t+\tilde{t}}{2}$ has the required properties. □

[40] As anthropologists have noted, exchange of favors works well in many societies without having exact accounts on who provided how much (e.g. Humphrey, 1985). Two important features of such societies are that members interact repeatedly and that the network of interactions fosters cooperation (e.g. Coleman and Coleman, 1994; Jackson et al., 2012). Indeed, we find indications for both features in our data set.

A.2. Proof of Lemma 1

In the Walrasian equilibrium allocation x^* for all agents i consuming a positive amount of good h we have $mu^i_h(x^{i,*}_h) = p^*_h$. Now, suppose $p^*_h \geq p_h$. Then $mu^i_h(x^{i,*}_h) \geq p_h$ for every i consuming a positive amount of good h at the price p^*. Since $mu^i_h(\hat{x}^i_h) = p_h$, $x^{i,*}_h \leq \hat{x}^i_h$ by concavity of u^i_h. Moreover, all agents consuming a positive amount of h at price p^* will do so at price $p_h \leq p^*_h$. Thus, $\sum_{i\in N} \hat{x}^i_h \geq \sum_{i\in N} x^{i,*}_h = \omega^h_h$, where the last equality holds because in the Walrasian equilibrium markets clear. Now, suppose $p^*_h < p_h$, then, for the analogous reasons as above, $\sum_{i\in N} \hat{x}^i_h < \sum_{i\in N} x^{i,*}_h = \omega^h_h$.

A.3. Proof of Proposition 1

We prove both statements separately.

(a) Non-scarce good h: Consider an allocation $\tilde{x}$ that does not satisfy this property. Hence, there is a buyer i and a good $h \neq i$ such that $\tilde{x}^i_h \neq \hat{x}^i_h$.
Suppose first $\tilde{x}^i_h > \hat{x}^i_h$, i.e. i receives more than desired. Then $\underline{Z}^i_h \leq 0 \leq \hat{x}^i_h < \tilde{x}^i_h \leq \bar{Z}^i_h$ (for the canonical constraints, the first and the last inequalities are equalities). Hence, within the constraints and within i's budget set, i could also reduce the amount that he buys from good h to $\tilde{x}^i_h - \epsilon$, and save ϵ of good 0 instead. By concavity $mu^i_h(\tilde{x}^i_h) < mu^i_h(\hat{x}^i_h) \leq p_h$, while the numeraire good has marginal utility of 1.[41] Thus, $MRS^i_{h,0}(\tilde{x}^i) = \frac{mu^i_h(\tilde{x}^i_h)}{mu^i_0(\tilde{x}^i_0)} < \frac{p_h}{1}$ and hence $\tilde{x}$ violates voluntariness (V).
Suppose second $\tilde{x}^i_h < \hat{x}^i_h$, i.e. i receives less than desired. Then he is constrained in market h, $\hat{x}^i_h > \tilde{x}^i_h = \bar{Z}^i_h$ (the last equality follows from feasibility and voluntariness). By concavity $mu^i_h(\tilde{x}^i_h) > mu^i_h(\hat{x}^i_h) \geq p_h$ ($\hat{x}^i_h = 0$ is not possible since $\tilde{x}^i_h < \hat{x}^i_h$), while the numeraire good has marginal utility of 1. Since $\sum_{i\in N} \tilde{x}^i_h = \omega^h_h > \sum_{i\in N} \hat{x}^i_h$ (the inequality is due to the fact that h is a non-scarce good), there must be an agent j with $\tilde{x}^j_h > \hat{x}^j_h$. If $j \neq h$, then $\tilde{x}$ violates voluntariness with respect to agent j as shown above (for agent i). Hence, consider the case that $j = h$. $\tilde{x}^h_h > \hat{x}^h_h$ means that the seller sells less than desired because $mu^h_h(\tilde{x}^h_h) < mu^i_h(\hat{x}^h_h) \leq p_h$ by concavity. Thus, $\hat{x}^h_h - \omega^h_h < \tilde{x}^h_h - \omega^h_h = \underline{Z}^h_h$ (the last equality follows from feasibility and voluntariness), i.e. the seller is constrained from selling more. This is a violation of weak order (WO). Indeed for t such that $t^i_h = \tilde{x}^i_h + \epsilon$ and $t^h_h = \tilde{x}^h_h - \epsilon - \omega^h_h$ and $t^i_0 = \tilde{x}^i_0 - \epsilon p_h$ and $t^h_0 = \tilde{x}^h_0 + \epsilon p_h$ and otherwise t fully corresponding to $\tilde{x}$, we have $t^i \succ^i \tilde{x}^i - \omega^i$ and $t^h \succ^h \tilde{x}^h - \omega^h$ and $(t^i_h - \bar{Z}^i_h)(t^h_h - \underline{Z}^h_h) = \epsilon \cdot (-\epsilon) < 0$.

(b) Scarce good h: Consider an allocation $\tilde{x}$ that does not satisfy this property. Suppose first that for some $i \neq h$, $\tilde{x}^i_h > \hat{x}^i_h$. This is a violation of voluntariness (V) as shown in the proof above.[42] From now on assume that $\forall i \neq h$, $\tilde{x}^i_h \leq \hat{x}^i_h$ and $\tilde{x}^h_h \neq \hat{x}^h_h$.
Suppose first $\tilde{x}^h_h < \hat{x}^h_h$, i.e. h sells more than desired. Then $\underline{Z}^h_h \leq \tilde{x}^h_h - \omega^h_h < \hat{x}^h_h - \omega^h_h \leq 0 \leq \bar{Z}^h_h$. Hence, within the constraints and within h's budget set, h could also reduce the amount that she sells from her good h and consume more herself, $\tilde{x}^h_h + \epsilon$, in exchange for a smaller amount of good 0. By concavity $mu^h_h(\tilde{x}^h_h) > mu^h_h(\hat{x}^h_h) \geq p_h$, while the numeraire good has marginal utility of 1. Thus, $\tilde{x}$ violates voluntariness (V).
Suppose second $\tilde{x}^h_h > \hat{x}^h_h$, i.e. h sells less than desired. Then she is constrained in market h, i.e. $\hat{x}^h_h - \omega^h_h < \tilde{x}^h_h - \omega^h_h = \underline{Z}^h_h$ (the last equality follows from feasibility and voluntariness). By concavity $mu^h_h(\tilde{x}^h_h) < mu^h_h(\hat{x}^h_h) \leq p_h$, while the numeraire good has marginal utility of 1. Since $\sum_{i\in N} \tilde{x}^i_h = \omega^h_h \leq \sum_{i\in N} \hat{x}^i_h$ (the inequality is due to the fact that h is a scarce good), there must be an agent i with $\tilde{x}^i_h < \hat{x}^i_h$, i.e. who buys less than desired. By concavity $mu^i_h(\tilde{x}^i_h) > mu^i_h(\hat{x}^i_h) \geq p_h$. Thus, (by feasibility and voluntariness) $\bar{Z}^i_h = \tilde{x}^i_h < \hat{x}^i_h$, i.e. buyer i is constrained from buying more. This is a violation of weak order (WO). Indeed for t such that $t^i_h = \tilde{x}^i_h + \epsilon$ and $t^h_h = \tilde{x}^h_h - \epsilon - \omega^h_h$ and $t^i_0 = \tilde{x}^i_0 - \epsilon p_h$ and $t^h_0 = \tilde{x}^h_0 + \epsilon p_h$ and otherwise t fully corresponding to $\tilde{x}$, we have $t^i \succ^i \tilde{x}^i - \omega^i$ and $t^h \succ^h \tilde{x}^h - \omega^h$ and $(t^i_h - \bar{Z}^i_h)(t^h_h - \underline{Z}^h_h) = \epsilon \cdot (-\epsilon) < 0$.

A.4. Proof of Proposition 2

Proof. There are two assertions to prove.

1. Pareto efficiency: Suppose good h is non-scarce and $\hat{x}^i_h > 0$ where $i \neq h$. Proposition 1 directly implies that in any FPE x: $MRS^h_{h,0}(x^i) < p_h$ and $MRS^i_{h,0}(x^i) = p_h$. Since preferences are continuous, a Pareto improving trade, in which h sells some amount to i at a price slightly below p_h, must exist.
2. Constrained efficiency: By weak interiority, the number of non-scarce markets is not equal to one.

[41] Boundary solutions are covered by "$\leq$": $\hat{x}^i_h = 0$ is possible, but $\hat{x}^i_h = \omega^i_h$ not since $\hat{x}^i_h < \tilde{x}^i_h \leq \omega^i_h$.

[42] Indeed, then $\underline{Z}^i_h \leq 0 \leq \hat{x}^i_h < \tilde{x}^i_h \leq \bar{Z}^i_h$. Hence, within the constraints and within i's budget set, i could also reduce the amount that he buys from good h, $\tilde{x}^i_h - \epsilon$ and save ϵ of good 0 instead. By concavity $mu^i_h(\tilde{x}^i_h) < mu^i_h(\hat{x}^i_h) \leq p_h$, while the numeraire good has marginal utility of 1.

(a) Suppose the number of non-scarce markets is larger than one. Take any supplier i of a non-scarce good i. By Proposition 1, in equilibrium $x_i^i > \hat{x}_i^i$ and hence $mu_i^i(x_i^i) < p_i$. By assumption of weak interiority, there exists another non-scarce good h such that $\hat{x}_h^i > 0$, which implies that in equilibrium $mu_h^i(x_h^i) \geq p_h$. Taken together $hi \in R^i$, where the binary relation R^i is defined for a fixed allocation x and fixed prices p_j and p_k as follows: $jk \in R^i \Leftrightarrow x_k^i > 0$ and $MRS_{j,k}^i(x^i) > \frac{p_j}{p_k}$.[43] Denote $i = h_1$ and $h = h_2$. Since h_2 is non-scarce either, a good h_3 exists, such that $h_3h_2 \in R^{h_2}$. If $h_3 = h_1$, a Pareto improving chain exists. If $h_3 \neq h_1$, a good h_4 must exist such that $h_4h_3 \in R^{h_3}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5, and so on. Eventually at good h_{k+1} it must be that $h_{k+1} = h_1$ or $h_{k+1} = h_2$ or... or $h_{k+1} = h_{k-1}$; and we have found a Pareto improving chain.

(b) Suppose the number of non-scarce markets is zero. Take any market $h \neq 0$. The assumption $p_h^* \neq p_h$ implies $p_h^* > p_h$ for scarce goods (by Lemma 1). Since markets clear in Walrasian equilibrium and Walrasian prices are larger than fixed prices, there is at least one agent who is constrained from buying on this market. Hence, for each good $h \neq 0$, there is some agent i with $mu_h^i(x_h^i) > p_h$, while $mu_i^i(x_i^i) = p_i$ (by Proposition 1).

Now, consider any good h_1. By the argument above, there exists a good h_2 such that $mu_{h_1}^{h_2}(x_{h_1}^{h_2}) > p_{h_1}$, while $mu_{h_2}^{h_2}(x_{h_2}^{h_2}) = p_{h_2}$. Thus, $h_1h_2 \in R^{h_2}$. Likewise, for good h_2, there is a an agent h_3 and the corresponding good h_3 such that $mu_{h_2}^{h_3}(x_{h_2}^{h_3}) > p_{h_2}$, while $mu_{h_3}^{h_3}(x_{h_3}^{h_3}) = p_{h_3}$. Thus, $h_2h_3 \in R^{h_3}$. If $h_1 = h_3$, a Pareto improving chain exists. If $h_1 \neq h_3$, a good h_4 exists $mu_{h_3}^{h_4}(x_{h_3}^{h_4}) > p_{h_3}$, while $mu_{h_4}^{h_4}(x_{h_4}^{h_4}) = p_{h_4}$. Thus, $h_3h_4 \in R^{h_4}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5, and so on. Since there are n goods, eventually at good h_{n+1} it must be that $h_{n+1} = h_1$ or $h_{n+1} = h_2$ or... or $h_{n+1} = h_{n-1}$; and we have found a Pareto improving chain.

□

A.5. Proof of Proposition 3

There are two assertions to prove.

1. Suppose good h is scarce. By Lemma 1, $p_h^* \geq p_h$. Hence, the demand of agent h for her own good is lower under Walrasian prices than under fixed prices. She gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is scarce (by Proposition 1). Hence, $x_h^{h,*} \leq \hat{x}_h^h = x_h^h$. Thus, $\omega_h^h - \sum_{i \neq h} t_h^{i,*} = x_h^{h,*} \leq x_h^h = \omega_h^h - \sum_{i \neq h} t_h^i$, which yields the result.
2. Suppose h is non-scarce. By Lemma 1, $p_h^* < p_h$. Hence, the demand of all agents $i \neq h$ is larger under Walrasian prices than under fixed prices. Any agent $i \neq h$ gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is non-scarce (by Proposition 1). Hence, $x_h^{i,*} \geq \hat{x}_h^i = x_h^i$. Thus, $t_h^i = x_h^i \leq x_h^{i,*} = t_h^{i,*}$, $\forall i \neq h$.

A.6. Proof of Corollary 1

There are two assertions to prove.

(i) Suppose h is scarce. Then $y^{h,*} = \sum_{i \neq h} t_h^{i,*} \cdot p_h^* \geq \sum_{i \neq h} t_h^i \cdot p_h = y^h$ since by Proposition 3 $\sum_{i \neq h} t_h^{i,*} \geq \sum_{i \neq h} t_h^i$ and by Lemma 1 $p_h^* \geq p_h$.

(ii) Suppose h is non-scarce. We have to show $y^{h,*} \geq y^h$ if and only if $|\epsilon_h| \geq \frac{p_h}{p_h^*}$. Let us rewrite $\epsilon_h = \frac{Q_h^* - Q_h}{p_h^* - p_h} \cdot \frac{p_h}{Q_h}$ to have $Q_h^* = Q_h(1 + \frac{p_h^* - p_h}{p_h}\epsilon_h)$, which we plug into the following expression.

$$y^{h,*} - y^h \geq 0 \tag{A.1}$$

$$Q_h^* p_h^* - Q_h p_h \geq 0 \tag{A.2}$$

$$Q_h(1 + \frac{p_h^* - p_h}{p_h}\epsilon_h)p_h^* - Q_h p_h \geq 0 \tag{A.3}$$

[43] The binary relation R^i indicates which trades agent i would accept. $jk \in R^i$ has the interpretation that agent i is willing to give up a small amount of good k to receive $\frac{p_k}{p_j}$ times that amount of good j.

$$Q_h\left[p_h^* + \frac{p_h^* - p_h}{p_h}\epsilon_h p_h^* - p_h\right] \geq 0 \tag{A.4}$$

$$Q_h\left[(p_h^* - p_h)(1 + \frac{p_h^*}{p_h}\epsilon_h)\right] \geq 0 \tag{A.5}$$

$Q_h > 0$ by assumption. By Lemma 1 we have $p_h^* - p_h < 0$. Hence the inequality above holds if and only if $(1 + \frac{p_h^*}{p_h}\epsilon_h) \leq 0$, which is equivalent to $\epsilon \leq -\frac{p_h}{p_h^*}$ and to $|\epsilon_h| \geq \frac{p_h}{p_h^*}$ because $\epsilon_h < 0$.

A.7. Proof of Lemma 2

We have homogeneous agents, i.e. $\omega_i^i = w > 0$ for all $i \neq 0$ and $\omega_i^j = 0$ for $i \neq j$, as well as $U^i(x^i) = x_0^i + u(x_1^i) + \ldots + u(x_i^i) + \ldots + u(x_n^i)$ for each agent i.

When $\hat{x}_h^i(p_h) = \hat{x}(p_h)$ denotes the unrestricted demand of agent i for good $h \neq 0$ (and in fact for every agent and every good), we have a unique price $p_h^* > 0$ with $mu(\frac{w}{n}) = p_h^*$, which is market clearing, i.e. $n * \hat{x}(p_h^*) = w$, as all agents have the same demand. The Walrasian price $p^* = (1, p_1^*, \ldots, p_n^*)$ leads to the allocation $x^i(p^*, N) = (0, \frac{w}{n}, \ldots, \frac{w}{n})$ for every agent i. For $p_i \leq p_i^*$, good i is scarce such that agent i consumes the desired amount $\hat{x}(p) \geq \frac{1}{n}$ with $mu(\hat{x}(p)) = p_i$ and sells $w - \hat{x}(p)$ (by Proposition 1). For $p_i > p_i^*$, good i is non-scarce such that any agent $j \neq i$ receives the desired amount $\hat{x}(p) < \frac{1}{n}$ with $mu(\hat{x}(p)) = p_i$ and i consumes $\omega - (n-1)\hat{x}(p)$ (by Proposition 1).

$x(p^*, N)$ is a Pareto efficient allocation satisfying $mu(x_h^i) = mu(x_h^j) = p_h^*$ for all agents i, j and goods h. p^* is unique because for any price $\rho > p_i^*$ there is excess supply for good $i \neq 0$, and for any price $\rho < p_i^*$ there is excess demand for every good $i \neq 0$. In both cases $x_i^i > x_i^j$ for some $j \neq i$ (by Proposition 1) and a Pareto superior allocation exists since $mu(x_i^i) < mu(x_i^j)$, while $mu(x_0^i) = mu(x_0^j)$. Hence, there is no price $p = (1, \rho, \ldots, \rho)$ such that all agents are weakly better off than under price p^*. Considering that under p^* all agents receive the same payoff directly implies that each agent strictly prefers p^* over every other common price ρ.

Consider $\rho > p_i^*$. Then every good $i \neq 0$ is non-scare. By Proposition 1, for every $j \neq i$, $x_i^j = x_j^i < x_i^i = x_j^j$. For larger prices, either demand converges to zero (when $mu(0) = \infty$) or there is a threshold $\bar{\rho} := mu(0)$ such that for prices $\rho \geq \bar{\rho}$ demand is zero (in which case there is no trade and $U^i(x^i(p, N)) = u(w)$). Proposition 1 implies for every common price $\rho' \in (p_i^*, \rho)$, with $\rho < \bar{\rho}$ if applicable, that the resulting consumption bundle $x^i((1, \rho', \ldots, \rho'), N)$ is a convex combination between $x^i(p^*, N)$ and $x^i((1, \rho, \ldots, \rho), N)$. Concavity of u implies that agent i prefers any common price $\rho' \in (p_i^*, \rho)$ over ρ.

Consider $\rho < p_i^*$. Then every good $i \neq 0$ is strictly scarce. By Proposition 1, for every $j \neq i$, $x_i^j = x_j^i < x_i^i = x_j^j$ and for every common price $\rho'' \in (\rho, p_i^*)$ the resulting consumption bundle $x^i((1, \rho'', \ldots, \rho''), N)$ is a convex combination between $x^i(p^*, N)$ and $x^i((1, \rho, \ldots, \rho), N)$. Concavity of u implies that agent i prefers any common price $\rho'' \in (\rho, p_i^*)$ over ρ.

Taken together $U^i(x^i(1, \rho, \ldots, \rho), N)$ is increasing in ρ for $\rho < p_i^*$, maximal at $\rho = p_i^*$, decreasing for $p_i^* < \rho < \bar{\rho}$ and constant for $\rho \geq \bar{\rho}$ if applicable.

A.8. Proof of Observation 1

We are looking for subgame perfect Nash equilibria that satisfy the three selection criteria. (i) The platform treats all agents equally. That is, under fixed prices it chooses a common price $p_i =: \rho\ \forall i \neq 0$. (ii) Agents' strategies are symmetric. That is, for any platform strategy, all agents make the same participation decision, and choose the same price $p_i =: \rho$ in case of participation and flexible prices. (iii) Agents coordinate on participation. That is, in every subgame in which both, none participates and all participate, are equilibria, we select the full participation equilibrium.

When the platform has chosen fixed prices p and fee f in Stage I, agents in Stage II only decide whether to participate or not. In such a subgame it is always a symmetric Nash equilibrium that no agent participates. Indeed, deviation to participate as the only member leads to the same consumption utility as the outside option (because there is no trade), but additionally to fee f. The other symmetric strategy profile in the subgame is that all agents participate. This is a Nash equilibrium in the subgame if and only if $U^i(x^i(p, N)) - f \geq U^i(\omega^i)$.

When the platform has chosen flexible prices and fee f in Stage I, agents in Stage II decide simultaneously upon participation and price p_i in case of participation. In such a subgame there is again the trivial equilibrium that no agent participates. Indeed, a single player cannot improve by participating alone and setting some price p_i. The other type of symmetric strategy profile in the subgame is that all agents participate and all choose a certain price $p_i = \rho$. This is a Nash equilibrium in the subgame if and only if two conditions are met. First, no agent i prefers to deviate to not participate, which is satisfied if and only if $U^i\big(x^i(p, N)\big) - f \geq U^i(\omega^i)$. Second, no agent i prefers to participate but change the price from $p_i = \rho$ to some $p_i' \neq \rho$. It turns out that there is a unique price, $\rho > p_i^*$ for which this second condition holds, as we show next. Suppose first that $p_i \leq p_i^*$. Then good i is scarce and we have $U^i(x^i(p, N)) = p_i(w - \hat{x}(p_i)) + u(\hat{x}(p_i)) + \sum_{j \neq i, 0}\left(-p_j x_j^i + u(x_j^i)\right)$.[44] Ap-

[44] We could be more specific about the consumption of other goods, but these markets do not influence the optimal price on market i.

plying the chain rule yields $\frac{\partial U^i(x^i(p,N))}{\partial p_i} = w - \hat{x}(p_i) + \hat{x}'(p_i)[mu(\hat{x}(p_i)) - p_i]$ (reflecting that agent i changing price p_i affects her utility first by the change in revenue from selling and second by the change in utility from own consumption of the good i). Now since $mu(\hat{x}(p_i)) = p_i$, we observe that increasing the price increases utility whenever $w > \hat{x}(p_i)$. That is, whenever agent i actually sells some quantity, a price $p_i < p_i^*$ cannot be part of an equilibrium.

Suppose now that $p_i \geq p_i^*$. Then good i is weakly non-scarce and we have $U^i(x^i(p,N)) = p_i((n-1)\hat{x}(p_i)) + u(w-(n-1)\hat{x}(p_i)) + \sum_{j\neq i,0}\left(-p_j x_j^i + u(x_j^i)\right)$. Applying the chain rule yields $\frac{\partial U^i(x^i(p,N))}{\partial p_i} = (n-1)\hat{x}(p_i) + (n-1)\hat{x}'(p_i)[p_i - mu(w-(n-1)\hat{x}(p_i))]$. For $p_i = p_i^*$ the term in brackets is zero and hence utility is strictly increasing in p_i. Maximizing over p_i yields p_i^M which is characterized by the first order condition $\frac{\partial U^i(x^i(p,N))}{\partial p_i} = 0$, that is

$$p_i^M = mu(w-(n-1)\hat{x}(p_i^M)) + \frac{\hat{x}(p_i^M)}{-\hat{x}'(p_i^M)}. \tag{A.6}$$

Hence, choosing flexible prices in Stage I yields either the equilibrium in which no agent participates or the equilibrium in which all agents participate and choose $p_i = p_i^M$.[45] The latter occurs if and only if $U^i(x^i(p^M,N)) - f \geq U^i(\omega^i)$, where $p^M := (1, p_1^M, \ldots, p_n^M)$.

In Stage I, the platform anticipates the outcomes of all the subgames in Stage II. If it chooses the fee f too large, then there are only outcomes in which no agent participates. This cannot be optimal since with a fee $f = \epsilon$ sufficiently small and flexible prices the platform would gain $\Pi = \epsilon * n > 0$, as $U^i(x^i(p^M,N)) - \epsilon > U^i(\omega^i)$ (there are benefits from trade due to the concavity of the utility function). The optimal fee in the case of flexible prices satisfies $f = U^i(x^i(p^M,N)) - U^i(\omega^i) = p_i^M((n-1)\hat{x}(p_i^M)) + u(w-(n-1)\hat{x}(p_i^M)) + \sum_{j\neq i,0}\left(-p_j^M\hat{x}^j(p^M) + u(\hat{x}^j(p^M))\right) - u(w)$.

When choosing fixed prices, the platform could replicate flexible prices by setting prices $p = p^M$, but it chooses prices that maximize members' gross payoffs and sets the fine to absorb them. By Lemma 2, the gross payoff $U^i(x^i((1,\rho,\ldots,\rho),N))$ is maximized for $\rho = p_i^*$. The optimal fee is hence $f = U^i(x^i(p^*,N) - U^i(\omega^i) = nu(\frac{w}{n}) - u(w)$.

A.9. Proof of Observation 2

We have homogeneous logarithmic utility, as defined in Eq. (2), endowments $\omega_i^i = 1$ and $\omega_j^i = 0$ for all $i,j \in N$ with $i \neq j$, and prices $p = (1,\ldots,1)$.

The demand for every good $h \neq 0$ is $\hat{x}_h^i = \alpha_h - 1$. A good is scarce if and only if $n * \hat{x}_h^i \geq \omega_h^h$, which is hence equivalent to $\alpha_h \geq 1 + \frac{1}{n}$. Recall that $\alpha_h \in [1,2]$.

Now, we apply Proposition 1 to determine agent i's trades in the FPE x. Agent i sells $(n-1)(\alpha_i - 1)$ units if good i is non-scarce; and i sells $2-\alpha_i$ units if good i is scarce. Agent i buys $\alpha_h - 1$ units of good $h \neq i,0$ if good h is non-scarce; and i buys $\frac{2-\alpha_h}{n-1}$ units of good h if good h is scarce (based on equal rationing). This yields utility that consists of a selling related part and of a buying related part: $U^i(x^i) = U^i_{|selling} + U^i_{|buying}$, with

$$U^i_{|selling} := \begin{cases} (n-1)(\alpha_i - 1) + \alpha_i \log\left(2-(n-1)(\alpha_i-1)\right), & \text{if } \alpha_i \leq 1+\frac{1}{n} \\ 2-\alpha_i+\alpha_i\log(\alpha_i), & \text{if } \alpha_i > 1+\frac{1}{n} \end{cases}$$

and

$$U^i_{|buying} := \sum_{h\neq i,0,\alpha_h\leq 1+\frac{1}{n}} (1-\alpha_h+\alpha_h\log(\alpha_h)) + \sum_{h\neq i,0,\alpha_h>1+\frac{1}{n}} \left(-\frac{2-\alpha_h}{n-1} + \alpha_h \log(1+\frac{2-\alpha_h}{n-1})\right).$$

The outside option yields $U^i(\omega^i) = \alpha_i\log(1+1)$. Taken together, agent i's willingness to pay is $WTP^i(p,N) = U^i_{|selling} + U^i_{|buying} - U^i(\omega^i)$. Observe that $U^i_{|buying}$ is independent of α_i. Therefore,

$$\frac{\partial WTP^i(p,N)}{\partial\alpha_i} = \frac{\partial U^i_{|selling}}{\partial\alpha_i} - \frac{\partial U^i(\omega^i)}{\partial\alpha_i} = \frac{\partial U^i_{|selling}}{\partial\alpha_i} - \log(2). \tag{A.7}$$

For $\alpha_i > 1+\frac{1}{n}$ we get $\frac{\partial WTP^i(p,N)}{\partial\alpha_i} = \log(\alpha_i) - \log(2)$, which is strictly negative except if $\alpha_i = 2$. Hence, the willingness to pay for an agent with a scarce good is strictly decreasing in attractiveness. For $\alpha_i \leq 1+\frac{1}{n}$ we get $\frac{\partial WTP^i(p,N)}{\partial\alpha_i} = n-1+\log\left(n+1-\alpha_i(n-1)\right) + \frac{\alpha_i(n-1)}{\alpha_i(n-1)-n-1} - \log(2)$, which is negative for small α_i and positive for α_i close to $1+\frac{1}{n}$. Taken together, $WTP^i(p,N)$ is first increasing and then decreasing in α_i.

[45] Indeed, this price p_i^M coincides with the choice of a monopolist i who faces demand $(n-1)\hat{x}_i(p)$ and whose costs are the opportunity costs of his own good: $\max_{p_i} p_i(n-1)\hat{x}(p_i) - u^i(w-(n-1)\hat{x}(p_i))$.

A.10. Proof of Observation 3

We begin with Walrasian prices and then turn to monopoly prices.

In the Walrasian equilibrium with homogeneous utility function and endowments equal to 1 every agent i receives $x_h^{i,*} = \frac{1}{n}$ of every good $h \neq 0$.[46] Moreover, the Walrasian price of every good is $p_h^* = \alpha_h \frac{n}{n+1}$.

Hence, agent i sells $\frac{n-1}{n}$ units of good i and buys $\frac{1}{n}$ of every other good. This yields:

$$U^i(x^{i,*}) = \frac{n-1}{n}\alpha_i \frac{n}{n+1} + \alpha_i \log(1+\frac{1}{n}) + \sum_{h\neq i,0}\left(-\frac{1}{n}\alpha_h \frac{n}{n+1} + \alpha_h \log(1+\frac{1}{n})\right),$$

which simplifies to

$$\alpha_i\left(\frac{n-1}{n+1} + \log(1+\frac{1}{n})\right) + \left(-\frac{1}{1+n} + \log(1+\frac{1}{n})\right)\sum_{h\neq i,0}\alpha_h. \tag{A.8}$$

Using $U^i(\omega^i) = \alpha_i \log(1+1)$ and $WTP^i(p^*, N) = U^i(x^{i,*}) - U^i(\omega^i)$, we receive

$$\frac{\partial WTP^i(p^*, N)}{\partial \alpha_i} = \frac{n-1}{n+1} + \log(1+\frac{1}{n}) - \log(2), \tag{A.9}$$

which is strictly positive (for $n > 1$). Hence, an agent's willingness to pay for participating in a platform with Walrasian prices is linearly increasing in the own good's attractiveness.

Let us now turn to monopolistic pricing. Demand of a single consumer for good h is $\max\{\frac{\alpha_h}{p_h} - 1, 0\}$. Monopoly pricing solves

$$\max_{p_h \geq 0} p_h(n-1)[\frac{\alpha_h}{p_h} - 1] + \alpha_h \log\left(1 + 1 - (n-1)[\frac{\alpha_h}{p_h} - 1]\right).$$

This leads to price $p_h^M = \alpha_h z$, with $z := \frac{(n-1)+\sqrt{n^2+2n+5}}{2(n+1)}$.[47] Monopoly quantity of one agent is $x_h^{i,M} = \frac{\alpha_h}{\alpha_h z} - 1 = \frac{1}{z} - 1$, which is independent of α_h (for this utility function).

Hence, agent i sells $(n-1)(\frac{1}{z} - 1)$ units of good i and buys $\frac{1}{z} - 1$ of every other good. This yields:

$$U^i(x^{i,M}) = (n-1)(\frac{1}{z} - 1)\cdot \alpha_i z + \alpha_i \log\left(2 - (n-1)(\frac{1}{z} - 1)\right) - \sum_{h\neq i,0}(\frac{1}{z} - 1)\alpha_h z + \sum_{h\neq i,0}\alpha_h \log(\frac{1}{z}),$$

which simplifies to

$$U^i(x^{i,M}) = \alpha_i(n-1)(1-z) + \alpha_i \log\left(1 + n - \frac{1}{z}(n-1)\right) - (1-z)\sum_{h\neq i,0}\alpha_h + \log(\frac{1}{z})\sum_{h\neq i,0}\alpha_h.$$

Using $U^i(\omega^i) = \alpha_i \log(1+1)$ and $WTP^i(p^M, N) = U^i(x^{i,M}) - U^i(\omega^i)$, we receive

$$\frac{\partial WTP^i(p^M, N)}{\partial \alpha_i} = (n-1)(1-z) + \log\left(1 + n - \frac{1}{z}(n-1)\right) - \log(2), \tag{A.10}$$

which is strictly positive (for $n > 1$). Hence, an agent's willingness to pay for participating in a platform with monopoly prices is linearly increasing in the own good's attractiveness.

Appendix B. Extensions

B.1. More general endowment

We briefly discuss how our results change when we relax the assumption on the endowments, i.e. that every agent is endowed with only one good and that the number of goods m must equal the number of agents n. Hence, there can now be many sellers of a good and an agent can sell many goods. We call every agent who is endowed with more than he desires, i.e. $\omega_h^j > \hat{x}_h^j$, *net supplier* of this good and all others *net demanders*.

Then the characterization of all FPE becomes:

Proposition B.1 (General Characterization)**.** *In every FPE, each good $h \neq 0$ is allocated as follows:*

1. *If h is non-scarce, every net demander receives the desired amount, while every net supplier receives at least the desired amount. That is: $\forall i$ with $\omega_h^i \leq \hat{x}_h^i$, we have $x_h^i = \hat{x}_h^i$; and $\forall j$ with $\omega_h^j > \hat{x}_h^j$, we have $x_h^j \geq \hat{x}_h^j$.*

[46] The agents' preferences are homogeneous as all agents have the same utility function (Eq. (2)), but the agents' goods are differently attractive.

[47] As $z > \frac{n}{n+1}$, we can observe $p_h^M > p_h^*$.

2. *If h is scarce, every net demander receives at most his desired amount, while the net suppliers keep (exactly) the desired amount. That is:* $\forall i$ *with* $\omega_h^i \leq \hat{x}_h^i$, *we have* $x_h^i \leq \hat{x}_h^i$; *and* $\forall j$ *with* $\omega_h^i > \hat{x}_h^i$, *we have* $x_h^j = \hat{x}_h^j$.

Proof. The proof is fully analogous to the proof of Proposition 1. □

As Proposition B.1 shows, the characterization of Proposition 1 generalizes to the set-up with more general endowments. Only the statement about net suppliers of non-scarce goods becomes weaker. Before, the excess supply was kept by the unique supplier. Now, the notion of FPE does not determine how the excess supply is allocated among the net suppliers. The other parts are identical to Proposition 1.

For the results on inefficiency (Proposition 2) and less trade (Proposition 3) this leads to some adaptions but does not change the substance.

B.2. More general preferences

In this section, we extend the model by relaxing the assumption that the utility function is quasi-linear. The more general utility function has the following form:

$$U^i(x^i) = u_0^i(x_0^i) + u_1^i(x_1^i) + \ldots + u_h^i(x_h^i) + \ldots + u_n^i(x_n^i),$$

with marginal utility $mu_h^i(x_h^i) > 0$, $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ and $\lim_{x_h^i \to \infty} mu_h^i(x_h^i) = 0$ for all $i, h \neq 0$ and x_h^i; the inequality $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ is strict for all $h \neq 0$. A simple characterization as in Proposition 1 is then no longer possible because demand and supply on each market may now depend on the allocation on all other markets. It is even possible that a scarce good "becomes non-scarce" in the sense that there is excess supply in the fixed price equilibrium; and vice versa. Since Proposition 1 was key to show inefficiency (Proposition 2) and less trade (Proposition 3), the question arises whether these results can be reestablished. The short answer is: yes, partially.

We can show first that for each scarce good i there must exist an agent j who would be willing to trade good i in exchange for his own good j (but not necessarily for good 0).

Lemma B.1. *If good h is strictly scarce, i.e.* $\sum_{i \in N} \hat{x}_h^i > \omega_h^h$, *then in any FPE x there is an agent j who would like to trade h in exchange for his own good, i.e.* $MRS_{h,j}^j(x^j) > \frac{p_h}{p_j}$.

Proof. We first show that the seller of the scarce good h, receives at least the desired amount, i.e. $x_h^h \geq \hat{x}_h^h$. Assume to the contrary that $x_h^h < \hat{x}_h^h$. By voluntariness (V), we then have $x_0^h < \hat{x}_0^h$. Again by voluntariness (V), this implies $x_k^h < \hat{x}_k^h$ for any good k. Thus, $x_h^h < \hat{x}_h^h$ implies $x_k^h < \hat{x}_k^h$ for any good k (including the numeraire). But then $p \cdot x < p \cdot \omega$. Hence, x cannot be an equilibrium allocation. Second, if h is strictly scarce, there must be an agent j such that $mu_h^j(\hat{x}_h^j) < mu_h^j(x_h^j)$. Together, we therefore have $p_h mu_j^j(x_j^j) \leq p_h mu_j^j(\hat{x}_j^j) = p_j mu_h^j(\hat{x}_h^j) < p_j mu_h^j(x_h^j)$. □

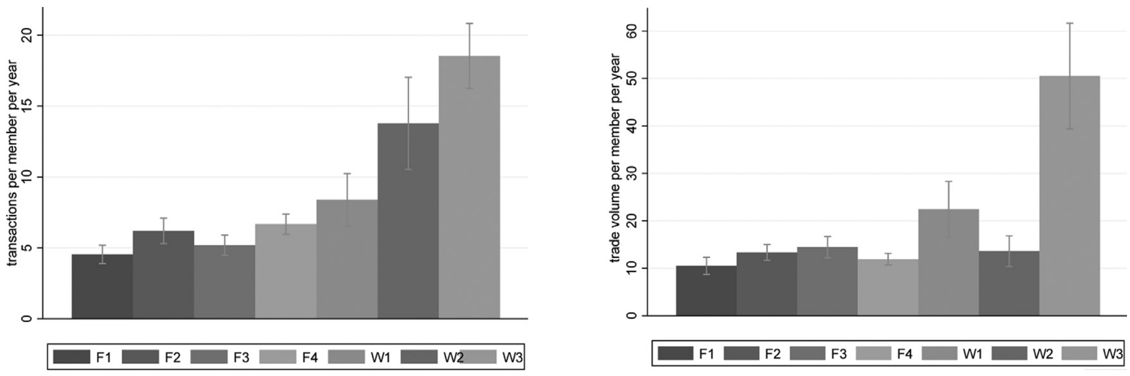
Lemma B.1 can be interpreted as follows: every (initially) scarce good remains "somewhat scarce." The main reason is that quantity constraints on the demand side can never increase supply. Hence, if there are two agents i and j, who both have a larger demand for the other's good than the other's (unconstrained) supply is, then they could improve in each FPE by mutual trade at the given price scheme. This leads to one kind of inefficiency that we establish in the following extension of Proposition 2.

Proposition B.2. *If there is a set of agents S such that their demand for their own goods exceeds the endowment, i.e.* $\forall i, h \in S$, $\sum_{i \in S} \hat{x}_h^i > \omega_h^h$, *then no FPE is constrained efficient.*

Proof. From Lemma B.1 we know that $\forall h \in S$, $x_h^h \geq \hat{x}_h^h$. Thus, for some $i \in S$, $x_h^i < \hat{x}_h^i$. This directly implies $hi \in R^i$ (where the binary relation R^i is defined as in the proof of Proposition 2), because $mu_h^i(\hat{x}_h^i) = \frac{p_h}{p_i} mu_i^i(\hat{x}_i^i)$ and $x_i^i \geq \hat{x}_i^i$ (again by Lemma B.1). At the same time there must exist an agent $j \neq i \in S$ such that $x_i^j < \hat{x}_i^j$. For the same reason as above $ij \in R^j$. We can continue as in the proof of Proposition 2 until we have found a Pareto improving chain. □

Hence, fixed prices often lead to constrained inefficient allocations even with more general preferences. We have shown this for one type of inefficiency (scarce goods, prices are "too low"), while for another (non-scarce goods, prices are "too high") the analogous result cannot be established. The reason is that quantity constraints on the demand side can easily increase demand for other goods. Hence, our inefficiency result, Proposition 2, partially extends to more general preferences.

Finally, we turn to our less trade result, Proposition 3. We can reuse the above mentioned fact that quantity constraints on the demand side can never increase supply to derive the following insight: *If the unconstrained supply of a scarce good h under the fixed price is lower than under a Walrasian price, then in every FPE the total amount traded of this good must be lower than in this Walrasian equilibrium.* In that sense, Proposition 3 extends to preferences that are not quasi-linear. Notice, however, that this insight is no longer generally valid, when we relax the assumption of additive separable preferences and therefore admit that the marginal rate of substitution between two goods depends on the consumption level of a third good.



(a) Number of Transactions (b) Trade Volume

Fig. C.1. Amount of trade: Panel (a) shows the average number of transactions per member per year. Panel (b) shows the average trade volume per member per year. Confidence intervals are standard 95% confidence intervals based on the heterogeneity between the members.

In that case, quantity constraints on the demand side can increase supply (consider, e.g., two complementary goods) and examples where trade volume of certain goods is higher in a FPE than in the Walrasian equilibrium can be constructed.

Our results hence partially extend to more general preferences. Importantly, the effects isolated in the special case of quasi-linear and additive separable preferences are still at work, they are in general simply accompanied by other potential effects.

Appendix C. Empirical background

C.1. Data set

We asked 18 platforms in Austria and Switzerland for their consent to analyze their transaction data and received a response of 55%, among whom the response was positive in 80% of the cases. One case with positive response was not considered because this data set did not even span one year. When obtaining the data, we agreed not to reveal the identity of these platforms.

All platforms in our data set have used the same payment software that records the transactions. This fact makes them directly comparable.

For each platform the recordings of the transactions begin with the introduction of the software. Our data set contains all transactions in the given period. For every transaction it shows the anonymous buyer, the anonymous seller and the total price paid in the platform-specific currency. Any member of a platform is anonymous, but uniquely identified across different transactions. Hence, our data set provides complete information about the transactions and about the trade network.

There are also several limitations of the data set. First, for many transactions the quantities are missing. Hence, we cannot measure the amount of trade in terms of quantities and we cannot investigate how prices per unit vary. Second, the description of goods and the categorization of goods is inconsistent across platforms such that it is hard to compare what

Table C.1
Description of pricing rules. The price recommendation is a literal translation from German. The currencies of W1-W3 have a platform-specific name.

ID	price recommendation	currency
F1	Performance is exchanged 1:1 – one hour of performance entitles to one hour of counter-performance.	hours
F2	An exchange rate of 1:1 is assumed. One hour of performance entitles to obtain one hour of performance for personal use.	hours
F3	The exchange among those willing to trade is accounted in hours and minutes.	hours
F4	Concerning the exchange of performance the following holds: Each hour has the same value.	hours
W1	Goods and services are generally traded according to currency hour-units.	hour-units
W2	The exchange partners determine the performance's value in currency units. As a point of reference, we recommend to value one hour working time by 100 currency units.	units
W3	We recommend to charge 100 currency units per hour. But two exchange partners decide on the price themselves.	units

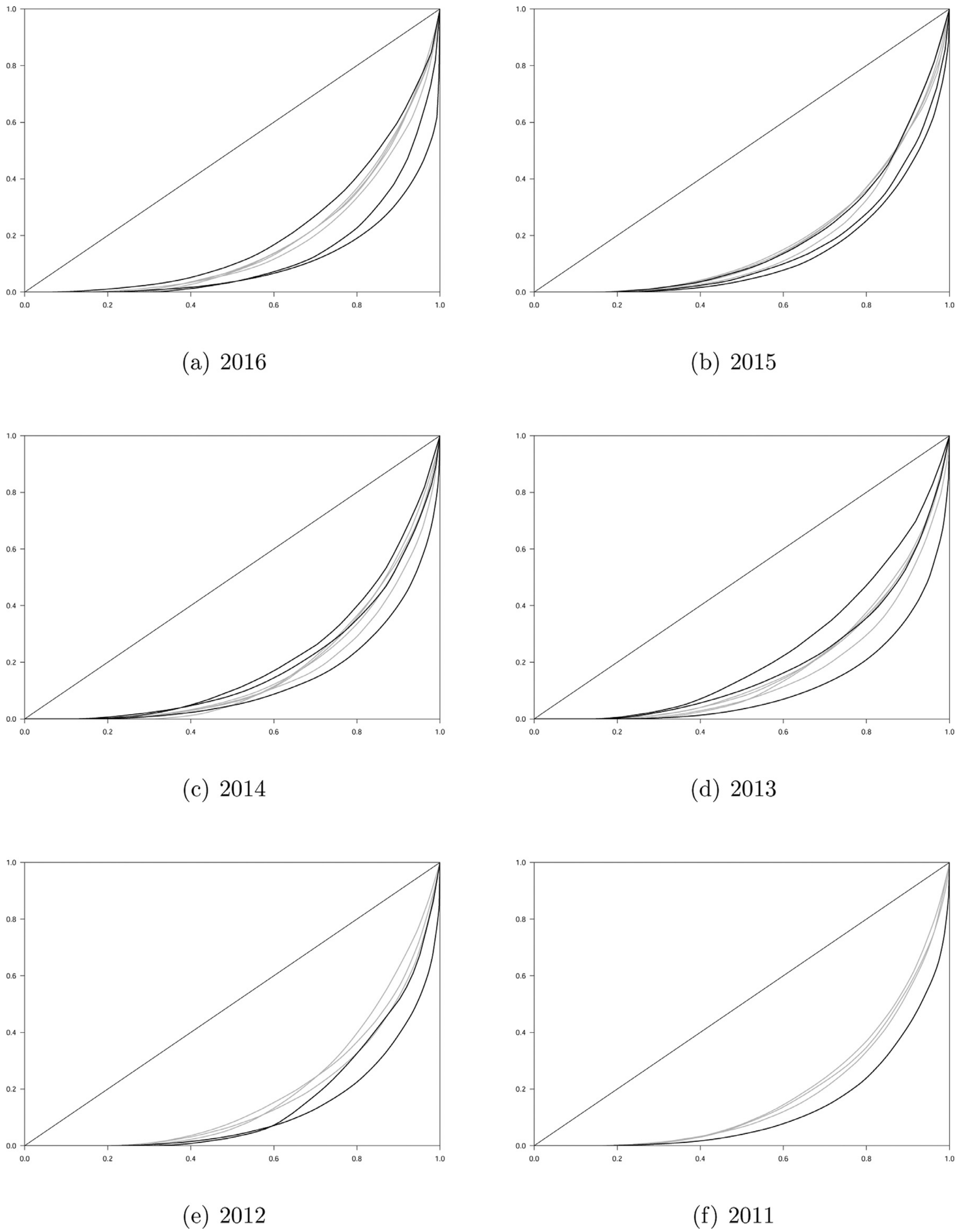
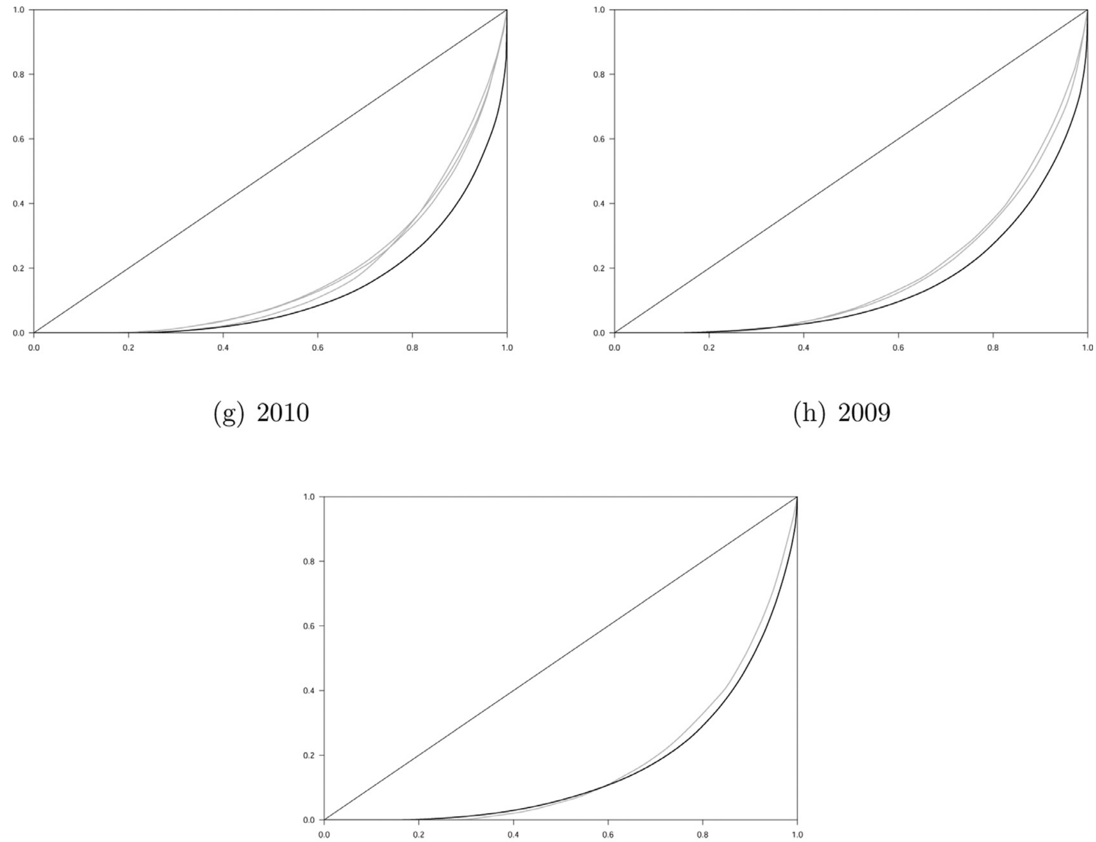


Fig. C.2. Lorenz curves of income distribution for each platform by year. The black lines illustrate inequality of platforms with flexible prices W1-W3 and the gray lines of those with fixed prices F1-F4.



(g) 2010

(h) 2009

(i) 2008

Fig. C.2. Continued

is traded. Some platforms explicitly emphasize exchange of services, while on all platforms both services and goods are admitted. Third, the data set does not contain member characteristics that could be compared across platforms. For instance, only for some platforms, there is information on age or gender or duration of membership; and there is no information about education or income. A few members were labeled as firms. As a test of robustness, we excluded all members that are identifiable as firms. This does not change any of the qualitative results.

C.2. Pricing rules

The main difference between platforms concerns the price setting, which is strictly restricted on platforms F1-F4 and more flexible on the other platforms W1-W3 (see Table C.1). Notice that using hours as currency fixes prices automatically since it ties each service to its duration, while using another currency gives much more flexibility. Hence, even if some platforms' price recommendations read similarly, the distinction between the platforms with fixed prices (F1-F4) and those with more flexible prices (W1-W3) is clear-cut. Other potentially relevant differences concern restrictions of the budget from below or above; and rules on how much to pay each year as a membership fee. In sum, it is however remarkable how similar the rules on these platforms are.

C.3. Amount of trade

We assessed the amount of trade by two complementary measures. The first measure is the number of transactions. The second measure is the trade volume, i.e. the money in the specific currency spent on trades (converted to hours in the case of W1-W3). Both measures are normalized by computing the amount per member per year to make the platforms comparable. More precisely, we compute for each member of a platform how much he traded on average per year for all the years that he was active, i.e. had at least one transaction, and average this number over all members. In this way we can account for the fact that individuals can join and leave a platform within the observed years. (Another normalization of

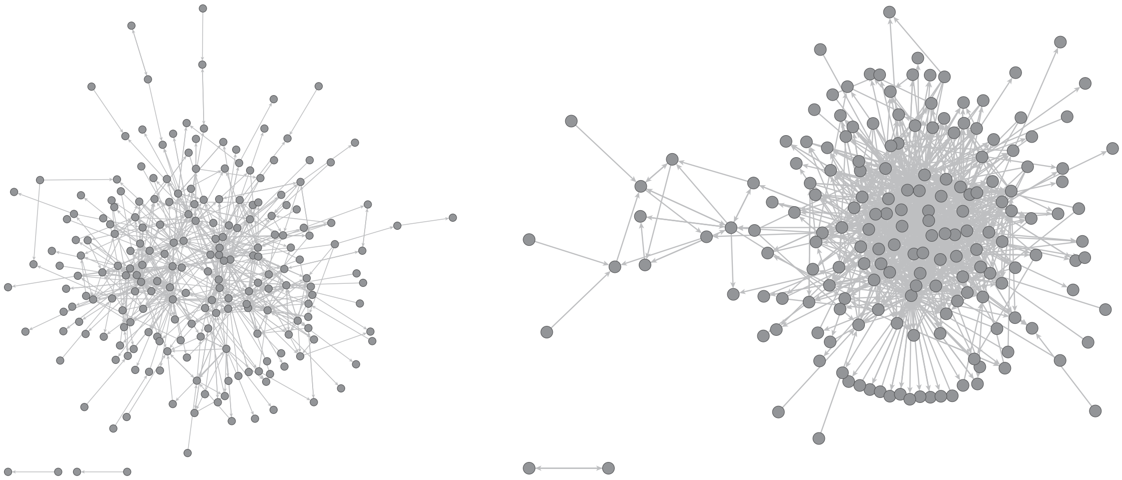


Fig. C.3. Trade network of platform F1 (panel (a)) and of platform W1 (panel (b)). Both networks are of similar age and of similar size (in terms of number of nodes), but the trade network of platform W1, the one with rather flexible prices, is much denser than the trade network of platform F1.

simply dividing the amount by the age of a platform and the number of members leads to an underestimation of the trade per member per year, but leads to the same qualitative differences between the platforms.)

The mean of both measures was reported in Table 2. Figure C.1 illustrates the means and the corresponding standard deviations.

C.4. Lorenz curves

Figure C.2 shows the Lorenz curve for each platform for each year. The id line is the benchmark of full equality. The other black lines illustrate inequality of platforms with flexible prices W1-W3 and the gray lines of those with fixed prices. Oftentimes, two black lines – corresponding to W1 and W3 – lie fully below all gray lines, which is known as Lorenz domination. When one distribution Lorenz dominates another one, then the first is more unequal with respect to most inequality measures.

C.5. Background on trade networks

Two trade networks are visualized by Fig. C.3. We illustrate the two youngest networks here because in the older networks there are so many trade relations that the visualization is not very informative. By convention, an arc from member i to some member j indicates that i bought a good from member j. The networks hence illustrate the flow of money.

In all networks there is one large component which consists of virtually all nodes. More precisely, taking the undirected network, in which every pair of customer and supplier is considered as linked trade partners, the number of nodes that are not connected to the largest component is below 2% in all networks.

Table 3 described several network statistics.[48] We provide here some more background on transitivity and the clustering coefficient. Transitivity is the usual notion for binary relations and hence measures how often the following implication holds: If i is a customer of j and j is a customer of k, then i must also be a customer of k. Transitivity (as all other measures in Table 3 except the clustering coefficient) uses the directed network of buyer-supplier relations. Concerning clustering, it seems at least as informative (and much more common) to use the undirected network of trade relationships to compute the clustering coefficient. Average clustering then answers the following question for an average member: How often are two of its trade partners, be it customers or suppliers, also trade partners themselves. More formally, a member's clustering coefficient is the number of trade relations in a member's neighborhood of trade partners divided by the number of possible trade relations in this neighborhood (Watts and Strogatz, 1998). We report the average clustering, while overall clustering, the fraction of complete triads over all triads with at least two links, would not paint a different picture here.

References

Abdulkadiroglu, A., Sönmez, T., 1999. House allocation with existing tenants. J. Econ. Theory 88 (2), 233–260. doi:10.1006/jeth.1999.2553.

Alesina, A., Angeletos, G.-M., 2005. Fairness and redistribution. Am. Econ. Rev. 95 (4), 960–980.

Almås, I., Cappelen, A.W., Sørensen, E.Ø., Tungodden, B., 2010. Fairness and the development of inequality acceptance. Science 328 (5982), 1176–1178. doi:10.1126/science.1187300.

[48] The network statistics are computed by the package *nwcommands* used in the software *STATA 14*.

Andersson, T., Cseh, Á., Ehlers, L., Erlanson, A., 2021. Organizing time exchanges: lessons from matching markets. Am. Econ. J.: Micro. 13 (1), 338–373.
Bakos, Y., Halaburda, H., 2019. Funding New Ventures with Digital Tokens: Due Diligence and Token Tradability. Technical Report. NYU Stern School of Business doi:10.2139/ssrn.3335650.
Belleflamme, P., Peitz, M., 2015. Industrial Organization: Markets and Strategies. Cambridge University Press.
Bénabou, R., Tirole, J., 2006. Belief in a just world and redistributive politics. Q. J. Econ. 121 (2), 699–746. doi:10.1162/qjec.2006.121.2.699.
Bergstrom, T., 2004. Experimental Markets and Chamberlin's Excess Trading Conjecture. University of California in Santa Barbara, Mimeo.
Buskens, V., 2002. Social Networks and Trust, vol. 30. Springer Science & Business Media.
Cahn, E., 2011. Time Banking: An Idea Whose Time has Come?. Yes Magazine. Retrieved 22 March 2018.
Chamberlin, E.H., 1948. An experimental imperfect market. J. Polit. Economy 56 (2), 95–108.
Coleman, J.S., 1994. Foundations of social theory. Belknap Series. Belknap Press of Harvard University Press. https://books.google.ch/books?id=a4Dl8tiX4b8C.
Coleman, J.S., 1988. Social capital in the creation of human capital. Am. J. Sociol. 94, 95–120.
Corominas-Bosch, M., 2004. Bargaining in a network of buyers and sellers. J. Econ. Theory 115 (1), 35–77. doi:10.1016/S0022-0531(03)00110-8.
Drèze, J., 1975. Existence of an equilibrium with price rigidity and quantity rationing. Int. Econ. Rev. 16 (2), 301–320.
Einav, L., Farronato, C., Levin, J., 2016. Peer-to-peer markets. Annu. Rev. Econom. 8, 615–635.
Freeman, L.C., 1978. Centrality in social networks conceptual clarification. Soc. Netw. 1 (3), 215–239.
Herings, P.J.-J., Konovalov, A., 2009. Constrained suboptimality when prices are non-competitive. J. Math. Econ. 45 (1), 43–58.
Humphrey, C., 1985. Barter and economic disintegration. Man 20 (1), 48–72.
Jackson, M.O., Rodriguez-Barraquer, T., Tan, X., 2012. Social capital and social quilts: network patterns of favor exchange. Am. Econ. Rev. 102 (5), 1857–1897.
Kranton, R.E., Minehart, D.F., 2001. A theory of buyer-seller networks. Am. Econ. Rev. 91 (3), 485–508.
Mailath, G.J., Postlewaite, A., Samuelson, L., 2016. Buying locally. Int. Econ. Rev. 57 (4), 1179–1200.
Maskin, E.S., Tirole, J., 1984. On the efficiency of fixed price equilibrium. J. Econ. Theory 32 (2), 317–327.
Rochet, J.-C., Tirole, J., 2003. Platform competition in two-sided markets. J. Eur. Econ. Assoc. 1 (4), 990–1029. doi:10.1162/154247603322493212.
Seyfang, G., 2003. Growing cohesive communities one favour at a time: social exclusion, active citizenship and time banks. Int. J. Urban Reg. Res. 27 (3), 699–706. doi:10.1111/1468-2427.00475.
Shapley, L., Scarf, H., 1974. On cores and indivisibility. J. Math. Econ. 1 (1), 23–37.
Sönmez, T., 1999. Strategy-proofness and essentially single-valued cores. Econometrica 67 (3), 677–689. http://www.jstor.org/stable/2999552.
Sweeney, J., Sweeney, R.J., 1977. Monetary theory and the great capitol hill baby sitting co-op crisis: comment. J. Money Credit Bank. 9 (1), 86–89. http://www.jstor.org/stable/1992001.
Warren, J., 1852. Equitable Commerce: A New Development of Principles. New York: Burt Franklin Press.
Watts, D.J., Strogatz, S.H., 1998. Collective dynamics of 'small-world' networks. Nature 393 (6684), 440.
Younés, Y., 1975. On the role of money in the process of exchange and the existence of a non-Walrasian equilibrium. Rev. Econ. Stud. 42 (4), 489–501.