## Supplementary Online Material

This supplementary online material belongs to the paper "The Strength of Weak Leaders: An Experiment on Social Influence and Social Learning in Teams" by Berno Buechel, Stefan Klößner, Martin Lochmüller, \& Heiko Rauhut. It consists of the following sections: ${ }^{1}$

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## B Mathematical Appendix

## B. 1 Theoretical Framework

The uncertainty is described by a probability space $(\Omega, \mathcal{F}, P)$, with $\Omega$ being the set of all states of nature, $\mathcal{F}$ being the $\sigma$-algebra of events, and $P$ being a probability measure on $\mathcal{F}$. For state of nature $\omega \in \Omega$, the correct answer to the question is denoted by $\theta(\omega)$, i.e., $\theta$ is a random variable on $(\Omega, \mathcal{F}, P)$. When the team members are confronted with the question, every team member $i$ is equipped with some information set describing $i$ 's knowledge about the true state of nature, $\mathcal{F}_{i}(0)$, with $i=1,2,3,4$ denoting the four team members. Thereby, $\mathcal{F}_{i}(0)$, technically a sub- $\sigma$-algebra of $\mathcal{F}$, contains all those events of which team member $i$ knows at time $t=0$ for sure whether they have occurred or not.

[^0]Building only on the information available to them at time $t=0$, all team members then state their guesses on the correct answer: we denote these answers at time $t=1$ by $X_{i}(1)$ ( $i=1,2,3,4$ ): the fact that team member $i$ can only make use of the information contained in $\mathcal{F}_{i}(0)$ technically translates into $X_{i}(1)$ being a random variable which must be $\mathcal{F}_{i}(0)$-measurable. Additionally, at time $t=1$, team member $i$ also provides information about the confidence level associated with $X_{i}(1)$ : this confidence statement will be denoted by $C_{i}(1)$, technically it is also a $\mathcal{F}_{i}(0)$-measurable random variable.

After the team members have stated their answers and confidence levels at time $t=1$, the team leader learns about the other team members' answers, $X_{2}(1), X_{3}(1)$, and $X_{4}(1)$, as well as their confidence levels, $C_{2}(1), C_{3}(1)$, and $C_{4}(1) .{ }^{2}$ Thus, the team leader can update by combining the initial information, $\mathcal{F}_{1}(0)$, and the observed answers and confidence levels of the other team members to build

$$
\mathcal{F}_{1}(1):=\sigma\left(\mathcal{F}_{1}(0), X_{2}(1), X_{3}(1), X_{4}(1), C_{2}(1), C_{3}(1), C_{4}(1)\right) \cdot \cdot^{3}
$$

Similarly, the non-central team members can update their information, however, they only observe the answer and confidence level stated by the team leader:

$$
\mathcal{F}_{i}(1):=\sigma\left(\mathcal{F}_{i}(0), X_{1}(1), C_{1}(1)\right), i=2,3,4 .
$$

Again, all team members $i$ now state their answers, $X_{i}(2)$, and confidence levels, $C_{i}(2)$. When stating these, team members can only build on the information set $\mathcal{F}_{i}(1)$, which however is in general larger than $\mathcal{F}_{i}(0)$, thus the answers and confidence levels stated at time $t=2$ may well differ from those stated at time $t=1$.

After the answers and confidence levels at time $t=2$ have been stated, the team leader again observes what the other team members have stated, which can be used for updating information:

$$
\begin{aligned}
\mathcal{F}_{1}(2) & :=\sigma\left(\mathcal{F}_{1}(1), X_{2}(2), X_{3}(2), X_{4}(2), C_{2}(2), C_{3}(2), C_{4}(2)\right) \\
& =\sigma\left(\left(\mathcal{F}_{1}(0), X_{i}(\tau), C_{i}(\tau), i=2,3,4, \tau=1,2\right) .\right.
\end{aligned}
$$

Similarly, the non-central team members can update their information, using the team leader's stated answer and confidence level:

$$
\mathcal{F}_{i}(2):=\sigma\left(\mathcal{F}_{i}(1), X_{1}(2), C_{1}(1)\right)=\sigma\left(\mathcal{F}_{i}(0), X_{1}(\tau), C_{1}(\tau), \tau=1,2\right), i=2,3,4 .
$$

Yet again, all team members $i$ now state their answers, $X_{i}(3)$, and confidence levels, $C_{i}(3)$. When stating these, team members can only build on the information set $\mathcal{F}_{i}(2)$, which however is in general larger than $\mathcal{F}_{i}(1)$, thus the answers and confidence levels stated at time $t=3$ may differ from those stated at time $t=2$. Afterwards, information updating takes place again, and the process of updating and stating answers and confidence levels goes on. Formally, this can

[^1]described by $X_{i}(t)$ and $C_{i}(t)$ being $\mathcal{F}_{i}(t-1)$-measurable for all team members $i=1,2,3,4$ and all times $t=1, \ldots, 6$, and
\[

$$
\begin{aligned}
\mathcal{F}_{1}(t) & :=\sigma\left(\mathcal{F}_{1}(t-1), X_{2}(t), X_{3}(t), X_{4}(t), C_{2}(t), C_{3}(t), C_{4}(t)\right) \\
& =\sigma\left(\left(\mathcal{F}_{1}(0), X_{i}(\tau), C_{i}(\tau), i=2,3,4, \tau=1, \ldots, t\right)\right.
\end{aligned}
$$
\]

as well as

$$
\mathcal{F}_{i}(t):=\sigma\left(\mathcal{F}_{i}(t-1), X_{1}(t), C_{1}(t)\right)=\sigma\left(\mathcal{F}_{i}(0), X_{1}(\tau), C_{1}(\tau), \tau=1, \ldots, t\right), i=2,3,4
$$

for all times $t=1, \ldots, 6$.
Using a payoff function, $\Pi$, which is decreasing in its argument, team member $i$ 's guess at time $t, X_{i}(t)$, is awarded by $\Pi\left(\left|\theta-X_{i}(t)\right|\right)$. In the end, the actual payoff is determined by randomly choosing the payoff belonging to one of the six answers, i.e., the payoff equals $\Pi\left(\left|\theta-X_{i}(1)\right|\right), \ldots, \Pi\left(\left|\theta-X_{i}(6)\right|\right)$, each with a probability of $1 / 6$.

## B. 2 Rational Models of Learning

Rational approaches assume that team members maximize their expected payoff. According to rational models, team member $i$ will choose $X_{i}(1), \ldots, X_{i}(6)$ and $C_{i}(1), \ldots, C_{i}(6)$ such that the expected payoff

$$
\frac{1}{6} \sum_{t=1}^{6} E\left(\Pi\left(\left|\theta-X_{i}(t)\right|\right)\right)
$$

becomes as large as possible.
First, we state an almost trivial lemma about the maximal amount of information the team members can collect.

Lemma B.1. Information acquisition in the team is bounded, no team member can learn more than the combination of all team members' initial information, technically:

$$
\mathcal{F}_{i}(t) \subseteq \sigma\left(\mathcal{F}_{1}(0), \mathcal{F}_{2}(0), \mathcal{F}_{3}(0), \mathcal{F}_{4}(0)\right)=: \mathcal{F}(0)
$$

We now discuss how the team leader is expected to behave under rational models of learning.
Proposition B.1. The following holds:

1. If the information contained in the pendants' first-round answers and confidence statements allows the team leader to get to know all of the information contained in the pendants' initial information that is important with respect to the correct answer, then the team leader will give the same, optimal answer in rounds 2 through 6. Formally,

$$
\begin{gathered}
\text { if } P\left(\theta \mid \sigma\left(\mathcal{F}_{1}(0), X_{i}(1), C_{i}(1), i=2,3,4\right)\right)=P(\theta \mid \mathcal{F}(0)) \text {, } \\
\text { then } X_{1}(t)=\underset{X \mathcal{F}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|)) \text { for } t=2, \ldots, 6 .
\end{gathered}
$$

This is in particular fulfilled if the team leader is able to completely infer the maximally available information, $\mathcal{F}(0)$, from the other team members' first round answers and confidence statements, i.e., if $\sigma\left(\mathcal{F}_{1}(0), X_{i}(1), C_{i}(1), i=2,3,4\right)=\mathcal{F}(0)$.
2. If $P\left(\theta \mid \sigma\left(\mathcal{F}_{1}(0), X_{i}(1), C_{i}(1), i=2,3,4\right)\right)=P(\theta \mid \mathcal{F}(0))$ (as in '1.'), then the team leader's optimal behavior is to give the answers $X^{*}:=\underset{X \mathcal{F}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|))$ in rounds $t=2, \ldots, 6$ and $\underset{X \mathcal{F}_{1}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|))$ in the first round.

Proof. We prove both parts separately.

1. Because of Lemma B.1, the team leader can never give an answer better than

$$
\underset{X \mathcal{F}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|)) .
$$

On the other hand, given that

$$
P\left(\theta \mid \sigma\left(\mathcal{F}_{1}(0), X_{i}(1), C_{i}(1), i=2,3,4\right)\right)=P(\theta \mid \mathcal{F}(0)),
$$

the team leader can form this conditional expectation at times $t=2, \ldots, 6$, because it can be formed when knowing $\mathcal{F}_{1}(0), X_{2}(1), X_{3}(1), X_{4}(1), C_{2}(1), C_{3}(1)$, and $C_{4}(1)$.
2. The statement for rounds 2 through 6 has already been proven in ' 1 .', and the statement for the first round follows from the same reasons. As this strategy separately maximizes each of the terms in the expected payoff, $\frac{1}{6} \sum_{t=1}^{6} E\left(\Pi\left(\left|\theta-X_{1}(t)\right|\right)\right)$, it is the optimal strategy for the team leader.

We now discuss how the pendants are expected to behave under rational models of learning.
Proposition B.2. The following holds:

1. If, from the team leader's answers and confidence statements in the first two rounds, pendant $i$ can learn everything that is relevant with respect to the correct answer, then pendant $i$ will state the optimal answer in rounds 3 through 6. Formally,

$$
\begin{aligned}
& \text { if } P\left(\theta \mid \sigma\left(\mathcal{F}_{i}(0), X_{1}(1), C_{1}(1), X_{1}(2), C_{1}(2)\right)\right)=P(\theta \mid \mathcal{F}(0)) \text {, } \\
& \text { then } X_{i}(t)=\underset{X \mathcal{F}(0) \text {-measurable }}{\arg \max } E(\Pi(|\theta-X|)) \text { for } t=3, \ldots, 6 \text {. }
\end{aligned}
$$

This is in particular fulfilled if pendant $i$ is able to completely infer the maximally available information, $\mathcal{F}(0)$, from the team leader's first and second round answers and confidence statements, i.e., if $\sigma\left(\mathcal{F}_{i}(0), X_{1}(1), C_{1}(1), X_{1}(2), C_{1}(2)\right)=\mathcal{F}(0)$.
2. If $P\left(\theta \mid \sigma\left(\mathcal{F}_{i}(0), X_{1}(1), C_{1}(1), X_{1}(2), C_{1}(2)\right)\right)=P(\theta \mid \mathcal{F}(0))$ (as in '1.'), then pendant $i$ 's optimal strategy is to give the answers $\underset{X \mathcal{F}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|))$ in rounds $t=3, \ldots, 6$,
$\underset{X \mathcal{F}_{i}(1)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|))$ in the second round, and $\underset{X \mathcal{F}_{i}(0)-\text { measurable }}{\arg \max } E(\Pi(|\theta-X|))$ in the first round.

Proof. The proofs are analogous to the corresponding proofs of Proposition B.1.
Overall, we have thus derived the following results which correspond to Prediction 1: if answers and confidence statements of the team members can be used to gain all relevant information contained in the team members' initial information, then the team leader will state the optimal answer in rounds 2 through 6 and the pendants will state the optimal answer in rounds 3 through 6.

## B. 3 Naïve Models of Learning

Naïve models of learning suppose that, from round to round, answers are convex combinations of own and other team members' answers according to weights $g_{i j}$ :

$$
\begin{aligned}
X_{1}(t+1)= & g_{11} X_{1}(t)+g_{12} X_{2}(t)+g_{13} X_{3}(t)+g_{14} X_{4}(t) \\
& X_{i}(t+1)=g_{i 1} X_{1}(t)+g_{i i} X_{i}(t), i=2,3,4
\end{aligned}
$$

Using the notation $X(t):=\left(X_{1}(t), \ldots, X_{4}(t)\right)^{\prime}$ for $t=1, \ldots, 6$, the updating can conveniently be written in vector and matrix notation as $X(t+1)=G X(t)$, where $G$ is given as follows:

$$
G=\left(\begin{array}{cccc}
g_{11} & g_{12} & g_{13} & g_{14}  \tag{B.1}\\
g_{21} & g_{22} & 0 & 0 \\
g_{31} & 0 & g_{33} & 0 \\
g_{41} & 0 & 0 & g_{44}
\end{array}\right)
$$

$G$ is a row-stochastic matrix which means that all entries of $G$ are non-negative and that, for each row, the sum of the corresponding entries equals unity. Additionally, to avoid trivial special cases, we assume that all the parameters in equation (B.1) are strictly positive: $g_{11}, g_{1 i}, g_{i 1}, g_{i i}>$ 0 for $i=2,3,4$, meaning that, when updating, the team leader takes into account the previous guesses of all team members, while all other team members update their guesses using their own and the team leader's previous guess. ${ }^{4}$

We first discuss under which conditions the team leader and pendants update their guesses only once and twice, respectively.

Proposition B.3. The following holds:

1. The team leader's guess is updated only once if and only if all team members put identical weights to the team leader when updating. Formally: $(1,0,0,0)^{\prime} G=(1,0,0,0)^{\prime} G^{t}$ for all $t$ if and only if $g_{11}=g_{21}=g_{31}=g_{41}$.

[^2]2. If the team leader's guess is updated only once, then the other team members update their guesses more than twice.

Proof. We prove both parts separately.

1. First of all, notice that the team leader's guess at time $t$ is given by

$$
(1,0,0,0)^{\prime} G^{t-1} X(1)
$$

Furthermore, if $(1,0,0,0)^{\prime} G=(1,0,0,0)^{\prime} G^{2}$, then $(1,0,0,0)^{\prime} G^{t-1}=(1,0,0,0)^{\prime} G$ for all $t$. We therefore only have to consider the first rows of $G$ and $G^{2}$. For $i=2,3,4$, the $i$-th element of the first row of $G^{2}$ is easily seen to be $g_{1 i}\left(g_{11}+g_{i i}\right)$. It equals the corresponding element of $G$ if and only if the equation $g_{1 i}=g_{1 i}\left(g_{11}+g_{i i}\right)$ holds. This is equivalent to $g_{11}+g_{i i}=1$ for $i=2,3,4$, which in turn is equivalent to $g_{1 i}=1-g_{i i}=g_{11}$ for $i=2,3,4$, because $G$ is row-stochastic.
2. Similar to above, for checking whether the team members update more than twice, it suffices to check whether the corresponding rows of $G^{2}$ equal those of $G^{3}$. We exemplarily consider the second team member and calculate $(0,1,0,0)^{\prime} G^{2}$ as well as $(0,1,0,0)^{\prime} G^{3}$, the calculations for $i=3,4$ are completely analogous. When the team leader updates only once, the row-stochastic matrix $G$ can be written as follows:

$$
G=\left(\begin{array}{cccc}
g_{11} & g_{12} & g_{13} & g_{14} \\
g_{11} & 1-g_{11} & 0 & 0 \\
g_{11} & 0 & 1-g_{11} & 0 \\
g_{11} & 0 & 0 & 1-g_{11}
\end{array}\right)
$$

with $g_{11}=1-g_{12}-g_{13}-g_{14}$. From this, we find:

$$
\begin{aligned}
& (0,1,0,0)^{\prime} G^{2}=\left(g_{11}, g_{12} g_{11}+\left(1-g_{11}\right)^{2}, g_{13} g_{11}, g_{14} g_{11}\right)^{\prime} \\
& (0,1,0,0)^{\prime} G^{3}=\left(g_{11}, g_{12} g_{11}\left(2-g_{11}\right)+\left(1-g_{11}\right)^{3}, g_{13} g_{11}\left(2-g_{11}\right), g_{14} g_{11}\left(2-g_{11}\right)\right)^{\prime}
\end{aligned}
$$

Thus, the second team member will in general state different guesses after the second and third updating.

Hence, we have a clear difference to the rational models. According to the naïve models, the team leader will update more than once, except for the special case that all pendants put the same weight on the team leader's previous guess when updating. If this special case is given, the pendants will update more than twice. Hence, under naïve models, it is not possible that agents state an optimal answer from round $t=3$ on.

We now turn our attention to the quality of learning, by studying the mean absolute error of the team members' guesses at the beginning and after the first round of communication.

Lemma B.2. Let $Y_{1}, \ldots, Y_{n}$ be $n$ random variables and denote the corresponding mean absolute errors by $\mathrm{MAE}_{Y_{i}}:=E\left(\left|Y_{i}-\theta\right|\right)$ for $i=1, \ldots, n$. Let further $Y:=\lambda_{1} Y_{1}+\ldots+\lambda_{n} Y_{n}$ be a convex combination of $Y_{1}, \ldots, Y_{n}$ with non-negative weights $\lambda$ summing to unity and denote the corresponding mean absolute error by $\mathrm{MAE}_{Y}:=E(|Y-\theta|)$. Then we have:

$$
\begin{equation*}
\operatorname{MAE}_{Y} \leq \lambda_{1} \operatorname{MAE}_{Y_{1}}+\ldots+\lambda_{n} \operatorname{MAE}_{Y_{n}} \tag{B.2}
\end{equation*}
$$

with equality if and only if $P\left(\left(Y_{1} \geq \theta \wedge \ldots \wedge Y_{n} \geq \theta\right) \vee\left(Y_{1} \leq \theta \wedge \ldots \wedge Y_{n} \leq \theta\right)\right)=1$.
Proof. First of all

$$
\begin{aligned}
|Y-\theta| & =\left|\lambda_{1} Y_{1}+\ldots+\lambda_{n} Y_{n}-\theta\right| \\
& =\left|\lambda_{1}\left(Y_{1}-\theta\right)+\ldots+\lambda_{n}\left(Y_{n}-\theta\right)\right| \\
& \leq \lambda_{1}\left|Y_{1}-\theta\right|+\ldots+\lambda_{n}\left|Y_{n}-\theta\right|
\end{aligned}
$$

from which taking expectations yields

$$
\operatorname{MAE}_{Y} \leq \lambda_{1} E\left(\left|Y_{1}-\theta\right|\right)+\ldots+\lambda_{n} E\left(\left|Y_{n}-\theta\right|\right)=\lambda_{1} \mathrm{MAE}_{Y_{1}}+\ldots+\lambda_{n} \mathrm{MAE}_{Y_{n}}
$$

with equality if and only if $\left|\lambda_{1}\left(Y_{1}-\theta\right)+\ldots+\lambda_{n}\left(Y_{n}-\theta\right)\right|$ equals $\lambda_{1}\left|Y_{1}-\theta\right|+\ldots+\lambda_{n}\left|Y_{n}-\theta\right|$ almost surely. Thus, to complete the proof, we only have to show that the latter happens if and only if $a_{1}:=Y_{1}-\theta, \ldots, a_{n}:=Y_{n}-\theta$ are either all non-negative or all non-positive almost surely. To this end, we compute $\left|a_{1}+\ldots+a_{n}\right|^{2}=\left(a_{1}+\ldots+a_{n}\right)^{2}$ and compare this quantity to $\left(\left|a_{1}\right|+\ldots+\left|a_{n}\right|\right)^{2}$. For the first quantity, we find $a_{1}^{2}+\ldots+a_{n}^{2}+\sum_{i \neq j} a_{i} a_{j}$, while the second quantity equals $a_{1}^{2}+\ldots+a_{n}^{2}+\sum_{i \neq j}\left|a_{i}\right|\left|a_{j}\right|$. The two quantities are thus equal if and only if $a_{i} a_{j}=\left|a_{i} a_{j}\right|$ for all $i, j$, which only happens if $a_{1}, \ldots, a_{n}$ are either all non-negative or all non-positive.

The following lemma is an immediate consequence of Lemma B.2.
Lemma B.3. For the weighted average $X_{2: 4}(1):=\lambda_{2} X_{2}(1)+\lambda_{3} X_{3}(1)+\lambda_{4} X_{4}(1)$ of the nonleaders' opinions $X_{2}(1), X_{3}(1)$, and $X_{4}(1)$, with $\lambda_{i}:=\frac{g_{1 i}}{g_{12}+g_{13}+g_{14}}$ for $i=2,3,4$, we have:

$$
\begin{equation*}
\operatorname{MAE}_{2: 4}(1) \leq \lambda_{2} \operatorname{MAE}_{2}(1)+\lambda_{3} \operatorname{MAE}_{3}(1)+\lambda_{4} \operatorname{MAE}_{4}(1), \tag{B.3}
\end{equation*}
$$

where $\operatorname{MAE}_{2: 4}(1):=E\left(\left|X_{2: 4}(1)-\theta\right|\right)$ and $\operatorname{MAE}_{i}(1):=E\left(\left|X_{i}(1)-\theta\right|\right)$ for $i=2,3$, 4. In equation (B.3), equality holds if and only if $X_{2}(1), X_{3}(1)$, and $X_{4}(1)$ lie on the same side of $\theta$ almost surely, i.e., if

$$
P\left(\left(X_{2}(1), X_{3}(1), X_{4}(1) \geq \theta\right) \vee\left(X_{2}(1), X_{3}(1), X_{4}(1) \leq \theta\right)\right)=1 .
$$

Lemma B. 3 shows that averaging the pendants' initial guesses typically is an improvement over their individual initial guesses.

Proposition B.4. The following holds:

1. If $\operatorname{MAE}_{2: 4}(1) \leq \operatorname{MAE}_{1}(1)$, then $\operatorname{MAE}_{1}(2) \leq \operatorname{MAE}_{1}(1)$, with equality if and only if $P\left(\left(X_{1}(1) \geq \theta \wedge X_{2: 4}(1) \geq \theta\right) \vee\left(X_{1}(1) \leq \theta \wedge X_{2: 4}(1) \leq \theta\right)\right)=1$.
2. For $i=2,3,4$ : if $\operatorname{MAE}_{1}(1) \leq \operatorname{MAE}_{i}(1)$, then $\operatorname{MAE}_{i}(2) \leq \operatorname{MAE}_{i}(1)$.

Proof. We prove both parts separately.

1. Since $X_{1}(2)=g_{11} X_{1}(1)+\left(g_{12}+g_{13}+g_{14}\right) X_{2: 4}(1)$ with $g_{11}+g_{12}+g_{13}+g_{14}=1$, Lemma B. 2 implies $\operatorname{MAE}_{1}(2) \leq g_{11} \operatorname{MAE}_{1}(1)+\left(1-g_{11}\right) \operatorname{MAE}_{2: 4}(1)$, from which the assertion follows immediately.
2. Applying Lemma B. 2 to $X_{i}(2)=g_{i 1} X_{1}(1)+g_{i i} X_{i}(1)$ yields $\operatorname{MAE}_{i}(2) \leq g_{i 1} \operatorname{MAE}_{1}(1)+$ $g_{i i} \operatorname{MAE}_{i}(1)$, from which the assertion follows immediately.

Proposition B. 4 shows that the team leader's guess will on average improve from the first to the second round if the combination of the other team members' initial guesses is a signal that is not worse than the team leader's initial one. This is a realistic assumption, particularly under the random treatment T0. Furthermore, a pendant's guess will improve after the first updating if the team leader's initial guess is on average not worse than that pendant's signal. This is a realistic assumption, particularly for treatments T1 accuracy and T2 confidence.

## B. 4 Specification and Extension of Rational Models

To specify the rational models, we assume that each agent's belief follows a beta distribution. This is a standard functional form for beliefs that live on intervals. ${ }^{5}$ With some assumptions on the distribution of signals, all agents' beliefs at any time indeed belong to the class of beta distributions. ${ }^{6}$ Assuming conditional independence of initial signals, Bayesian agents will state guesses that are convex combinations of their initial guesses. The weight on these guesses, however, depends on the signal quality of each agent $i$, which we denote by $n_{i}$. The model variations that we study differ in the assumptions about signal quality.

A baseline assumption is to suppose that the precision of each agent's signal is the same, i.e., $n_{i}=n_{j}$ for all $i, j$. In that case, the optimal guess $x^{*}$, which will be the consensus from round $t=3$ on, is simply the unweighted mean of the initial guesses $x_{i}(1)$. We call this the Standard Model. Alternatively, agents are assumed to communicate their belief fully by providing the guess and the confidence level. Then, for each answer $x_{i}(1)$ and its confidence $c_{i}(1)$, the center can determine the two parameters of the corresponding beta distribution and combine all initial beliefs in a rational manner, thereby updating leads to a combination of own and others' guesses - not with equal weights, but with larger weights for those guesses

[^3]which are tagged by high confidence. We call this the Sophisticated Model. Note that these are two opposing views on the informativeness of the confidence statement - either confidence is fully informative or confidence can be ignored - which lead to two models that both satisfy the requirements of Prediction 1, and are hence similar in most respects. They differ in their weighting of initial information.

The previous empirical literature on real people's beliefs and their updating finds two very strong and consistent patterns: overprecision and conservatism. ${ }^{7}$ There is a simple way to introduce both of them into our model: Agents overestimate their own signal precision by a factor $\tau_{i} \geq 1$; respectively, they underestimate the signal precision of the others by the inverse factor $\frac{1}{\tau_{i}}$. The motivation of this model variant is that overconfident agents suffer from overprecision in the sense that they perceive their signal as more precise than it is. ${ }^{8}$

Formally, this is a generalization of the Standard Model and the Sophisticated Model. This model also predicts that there are no more changes after $t=3$. However, this model does not predict consensus! The agents' opinions settle down in between the prediction of $x^{*}$ (i.e., the case $\tau_{i}=1$ for all $i \in N$ ) and their initial guess $x_{i}(1)$. The weight of the own initial guess is thereby increasing in overprecision $\tau_{i}$. In particular, if $\tau_{i} \rightarrow \infty$, then $x_{i}(t) \rightarrow x_{i}(1)$, i.e., infinitely overprecise agents are totally conservative and always stick to their initial guess. (We will include such a model as a baseline and call it the Sticking Model.)

To specify concrete models, we choose levels of overprecision $\tau_{i}$ that match with empirical results on overprecision. When asked for a $90 \%$ confidence interval, many people provide a $50 \%$ confidence interval instead. This is roughly induced by $\tau_{i}=5$. Incorporating conservatism of every agent into the Standard Model or, respectively, into the Sophisticated Model leads to the two models Standard-Plus Model and Sophisticated-Plus Model. In the Standard-Plus Model, agents behave very similarly to the Standard Model, but move only a fraction into the direction of the center, which corresponds to findings on conservatism. The only difference to the Sophisticated-Plus Model is simply that we specify the initial signal precision not as equal, but according to the confidence statements. Agents are assumed to know that others are overprecise and thus learn about the original signals by correcting for $\tau .{ }^{9}$

Importantly, the four models Standard Model, Sophisticated Model, Standard-Plus Model, and Sophisticated-Plus Model are all special cases of Bayesian models and hence produce the prediction that is formalized as Prediction 1. Except that, in the Standard-Plus Model and

[^4]the Sophisticated-Plus Model, agents do not state the same guess $x^{*}$ from round 3 on, but their subjectively perceived optimal guess $x_{i}^{*}$, which is a mixture between $x^{*}$ and the agent's initial guess $x_{i}(1)$. This difference is illustrated in Figure B. 1 below in the two left panels, which compare the dynamics of the Standard Model with the Standard-Plus Model in a simple example.

## B.4.1 Specific Rational Models

Building on the theoretical framework laid out above, we will now consider specific rational models, by specifying in particular what information team members initially possess. To start, however, we discuss how correct answers are modeled.

For ease of presentation, we interpret the correct answers to the questions asked in our experiment as points in $[0,1]$, although answers had to be integer numbers between 0 and 100 . For instance, 71, the correct answer to the question about the voter turnout to the federal elections in Germany in 2009, is translated into 0.71 and could also be interpreted as the probability of a randomly chosen eligible voter actually casting a ballot. This given, we assume that the prior, unconditional distribution of the correct answer is the uniform distribution on the unit interval: $\theta \sim U(0,1)$, with probability density function (pdf) $f_{\theta}(p)=1$ for all $p \in[0,1]$, meaning that, a priori, before any agent has received any information, all answers were equally likely to be the correct one. The uniform distribution corresponds to a beta distribution $\beta(1,1)$ which was originally suggested as the prior distribution by Thomas Bayes. With respect to initial information, we assume that each team member $i(i=1, \ldots, 4)$ observes a two-dimensional signal $\psi_{i}=\left(S_{i}, F_{i}\right)$ which is, conditional on $\theta$, stochastically independent from the other team members' signals. This signal can be interpreted in the following way: every team member $i$ has some pool of observations, where observations can either be 'successes' or 'failures', and the number of successes is $S_{i}$, while the number of failures is $F_{i}$. Here, 'successes' and 'failures' mean that the condition asked for is fulfilled or not: in case of the voter turnout, $S_{i}$ gives the number of people of which team member $i$ knows that they cast a vote, while $F_{i}$ denotes the number of people of which team member $i$ knows that they abstained from voting.

## The 'Standard' Model

In the 'Standard' model, it is assumed that all team members possess the same amount of information, i.e., that $S_{1}+F_{1}=\ldots=S_{4}+F_{4} \cdot{ }^{10}$ With respect to the link between the distribution of $\left(S_{i}, F_{i}\right)$ to the unknown, correct answer, we assume the following: the probability of observing $\left(S_{i}, F_{i}\right)=\left(s_{i}, f_{i}\right)$ is, conditional on $\theta=p$, given by ${ }^{11}\binom{s_{i}+f_{i}}{s_{i}} \cdot p^{s_{i}} \cdot(1-p)^{f_{i}}$. Put differently, the number of successes follows, conditional on $\theta=p$, a binomial distribution with parameters $p$ and $n:=s_{i}+f_{i}$. Observing the signal, team member $i$ may update the a priori

[^5]belief by using Bayes' rule, forming the distribution of $\theta$ conditional on observing $S_{i}=s_{i}$ :
$$
f_{\theta \mid S_{i}=s_{i}, F_{i}=f_{i}}(p)=\frac{\binom{s_{i}+f_{i}}{s_{i}} \cdot p^{s_{i}} \cdot(1-p)^{f_{i}}}{\int_{0}^{1}\binom{s_{i}+f_{i}}{s_{i}} \cdot \widetilde{p}^{s_{i}} \cdot(1-\widetilde{p})^{f_{i}} d \widetilde{p}}=\frac{p^{s_{i}} \cdot(1-p)^{f_{i}}}{B\left(s_{i}+1, f_{i}+1\right)},
$$
with $B(\alpha, \beta)$ denoting Euler's beta function. Thus, team member $i$ 's inital belief before communication is a beta distribution with parameters $s_{i}+1$ and $f_{i}+1$. Therefore, team member $i$ 's optimal answer in the first round is $p^{*}$, with $p^{*}$ maximizing
$$
\int_{0}^{1} \Pi\left(\left|p-p^{*}\right|\right) f_{\theta \mid S_{i}=s_{i}, F_{i}=f_{i}}(p) d p=\int_{0}^{1} \Pi\left(\left|p-p^{*}\right|\right) \frac{p^{s_{i}} \cdot(1-p)^{f_{i}}}{B\left(s_{i}+1, f_{i}+1\right)} d p
$$

Due to the specific structure of the payoff function used in our experiment, the beta distribution's mode, $\frac{s_{i}}{s_{i}+f_{i}}=\frac{s_{i}}{n}$, is a very good approximation to $p^{*}$, we will therefore assume that team member $i$ states the answer $X_{i}(1)=\frac{s_{i}}{s_{i}+f_{i}}=\frac{s_{i}}{n}$ in the first round. ${ }^{12}$ Continuing the example on voter turnout: If an individual knows about ten citizens that seven of them voted and three of them abstained, then his belief is beta distributed with a mode of $\frac{7}{7+3}=0.7$, which is his initial guess.

After the first round of answers, the team leader gets to know the answers of all team members, thus the team leader can easily recover $s_{2}, s_{3}$, and $s_{4}$ to gain the maximally available information, $\mathcal{F}(0)$. The corresponding belief upon maximal information, i.e., upon observing $s_{1}, \ldots, s_{4}$, is

$$
f_{\theta \mid S_{1}=s_{1}, \ldots, S_{4}=s_{4}, F_{1}=f_{1}, \ldots, F_{4}=f_{4}}(p)=\frac{p^{s_{1}+\ldots+s_{4}} \cdot(1-p)^{f_{1}+\ldots+f_{4}}}{B\left(1+\sum_{i=1}^{4} s_{i}, 1+\sum_{i=1}^{4} f_{i}\right)}
$$

again a beta distribution, with parameters $1+\sum_{i=1}^{4} s_{i}$ and $1+\sum_{i=1}^{4} f_{i}$. Thus, the team leader's optimal answer in rounds 2 through 6 is the corresponding mode, $\frac{s_{1}+\ldots+s_{4}}{s_{1}+\ldots+s_{4}+f_{1}+\ldots+f_{4}}=\frac{s_{1}+\ldots+s_{4}}{4 n}$, which can be rewritten as $\frac{1}{4} X_{1}(1)+\ldots+\frac{1}{4} X_{4}(1)$, an equally weighted average of the team members' first-round answers.

The other team members' second-round answers can be built using only the corresponding initial signal, $\psi_{i}$, as well as the team leader's first-round answer, $X_{1}(1)$. The latter allows to infer $s_{1}$, thus team member $i$ 's knowledge in the second round consists of $s_{1}$ and $s_{i}$. Analogously to above, it is easy to derive that the corresponding belief is again a beta distribution, with parameters $1+s_{1}+s_{i}$ and $1+f_{1}+f_{i}$. The corresponding optimal answers in the second round

[^6]thus are $\frac{1}{2} X_{1}(1)+\frac{1}{2} X_{i}(1)$, while from round 3 on, the team leader's optimal answer will be copied.

Overall, the updating in the 'Standard' model can be summarized as follows:
Summary 1 (Standard Model). The team leader computes the unweighted average of all team members' first-round answers and states $\frac{1}{4} X_{1}(1)+\ldots+\frac{1}{4} X_{4}(1)$ from round 2 on, the other team members state $\frac{1}{2} X_{1}(1)+\frac{1}{2} X_{i}(1)$ in round 2 , and they join the team leader in stating the average of the team's first-round answers from round 3 on.

## The 'Sophisticated' Model

For the 'Sophisticated' model, the link between the distribution of the signal $\psi_{i}=\left(S_{i}, F_{i}\right)$ to the unknown, correct answer, $\theta$, looks as follows: the probability of observing $\left(S_{i}, F_{i}\right)=\left(s_{i}, f_{i}\right)$ is, conditional on $\theta=p$ and $S_{i}+F_{i}=n_{i}:=s_{i}+f_{i}$, given by $\binom{n}{s_{i}} \cdot p^{s_{i}} \cdot(1-p)^{f_{i}}$. Put differently, the number of successes follows, conditional on $\theta=p$ and $S_{i}+F_{i}=n_{i}$, a binomial distribution with parameters $p$ and $n_{i}=S_{i}+F_{i}$. As for the 'Standard' model, one easily derives that team member $i$ 's belief about the correct answer is a beta distribution with parameters $1+s_{i}$ and $1+f_{i}$, implying that the first-round answer is $\frac{s_{i}}{s_{i}+f_{i}}=\frac{s_{i}}{n_{i}}$.

In our experiment, team members were not only asked about their guess with respect to the correct answer to the question at hand, but they also supplied a measure of the confidence in their answer. More precisely, they essentially provided an interval that should contain the correct answer with a probability of $90 \%$. Based on the $\operatorname{Beta}\left(1+s_{i}, 1+f_{i}\right)$-belief, team member $i$ 's first-round statement thus does not only consist of guessing the correct answer by $X_{i}(1)=\frac{s_{i}}{n_{i}}$, but also of supplying the corresponding confidence $C_{i}(1)$ which is a function of $s_{i}$ and $f_{i}, C_{i}(1)=\operatorname{Conf}\left(s_{i}, f_{i}\right)$.

After the first round of answers, the team leader gets to know not only the answers of all team members, $X_{i}(1)=\frac{s_{i}}{s_{i}+f_{i}}$, but also their confidence statements, $C_{i}(1)=\operatorname{Conf}\left(s_{i}, f_{i}\right)$ $(i=2,3,4)$. Using these, the team leader can recover $s_{2}, s_{3}$, and $s_{4}$ as well as $f_{2}, f_{3}$, and $f_{4}$, to gain the maximally available information, $\mathcal{F}(0)$. The corresponding belief upon maximal information, i.e., upon observing $s_{1}, \ldots, s_{4}, f_{1}, \ldots, f_{4}$, is

$$
f_{\theta \mid S_{1}=s_{1}, \ldots, S_{4}=s_{4}, F_{1}=f_{1}, \ldots, F_{4}=f_{4}}(p)=\frac{p^{s_{1}+\ldots+s_{4}} \cdot(1-p)^{f_{1}+\ldots+f_{4}}}{B\left(1+\sum_{i=1}^{4} s_{i}, 1+\sum_{i=1}^{4} f_{i}\right)},
$$

again a beta distribution, with parameters $1+\sum_{i=1}^{4} s_{i}$ and $1+\sum_{i=1}^{4} f_{i}$. Thus, the team leader's optimal answer in rounds 2 through 6 is the corresponding mode, $\frac{s_{1}+\ldots+s_{4}}{s_{1}+\ldots+s_{4}+f_{1}+\ldots+f_{4}}=\frac{s_{1}+\ldots+s_{4}}{n_{1}+\ldots+n_{4}}$, which can be rewritten as $\frac{n_{1}}{n_{1}+\ldots+n_{4}} X_{1}(1)+\ldots+\frac{n_{4}}{n_{1}+\ldots+n_{4}} X_{4}(1)$, a weighted average of the team members' first-round answers.

The other team members' second-round answers can be built using only the corresponding initial signal, $\psi_{i}$, as well as the team leader's first-round answer and confidence statement, $X_{1}(1)$ and $C_{1}(1)$. The latter quantities allow to infer $s_{1}$ and $f_{1}$, thus team member $i$ 's knowledge in the second round consists of $s_{1}, s_{i}$ and $f_{1}, f_{i}$. Analogously to above, it is easy to derive that the
corresponding belief is again a beta distribution, with parameters $1+s_{1}+s_{i}$ and $1+f_{1}+f_{i}$. The corresponding optimal answers in the second round thus are $\frac{s_{1}+s_{i}}{n_{1}+n_{i}}=\frac{n_{1}}{n_{1}+n_{i}} X_{1}(1)+\frac{n_{i}}{n_{1}+n_{i}} X_{i}(1)$, while from round 3 on, the team leader's optimal answer will be copied.

Overall, the updating in the 'Sophisticated' model can be summarized as follows:
Summary 2 (Sophisticated Model). The team leader computes a weighted average of all team members' first-round answers and states

$$
\frac{N_{1}}{N_{1}+N_{2}+N_{3}+N_{4}} X_{1}(1)+\ldots+\frac{N_{4}}{N_{1}+N_{2}+N_{3}+N_{4}} X_{4}(1)
$$

from round 2 on, the other team members state $\frac{N_{1}}{N_{1}+N_{i}} X_{1}(1)+\frac{N_{i}}{N_{1}+N_{i}} X_{i}(1)$ in round 2, and they join the team leader in stating the weighted average of the team's first-round answers from round 3 on.

## B.4.2 Models of Rational Learning with Conservatism

In the following, we will enrich the models of rational learning by conservatism that results from overprecision. Overprecision is the empirically observed phenomenon that people typically provide too narrow confidence intervals when asked about their confidence. To model overprecision, we will assume that agents treat their initial private signal as more precise than it actually is. We will further assume that agents account for the fact that other team members are overprecise, but are blind with respect to their own level of overprecision. While the level of overprecision is in principle agent-specific, our analysis focuses on the case in which all agents are equally overprecise. The models with overprecision nest the rational models when setting the level of overprecision to zero.

## The 'Standard Plus' model

By the 'Standard Plus' model, we denote the extension of the 'Standard' model by overprecision. The only difference to the 'Standard' model is that we assume that team members misinterpret their signal: when the signal actually is $\psi_{i}=\left(s_{i}, f_{i}\right)$, team member $i$ will interpret it as if the received signal was $\left(\tau s_{i}, \tau f_{i}\right)$, where $\tau \geq 1$ is a parameter to capture overprecision. ${ }^{13}$ Therefore, team member $i$ 's belief in the first round will be given by a $\operatorname{Beta}\left(1+\tau s_{i}, 1+\tau f_{i}\right)$ distribution, leading to the first-round answer $\frac{\tau s_{i}}{\tau s_{i}+\tau f_{i}}=\frac{s_{i}}{n}$, as in the 'Standard' model. However, after learning from the other team members, the second-round answers are still prone to overprecision: from round 2 on, the team leader will state $\frac{\tau s_{1}+s_{2}+\ldots+s_{4}}{\tau s_{1}+s_{2}+\ldots+s_{4}+\tau f_{1}+f_{2}+\ldots+f_{4}}$, which can be rewritten as $\frac{\tau}{\tau+3} X_{1}(1)+\frac{1}{\tau+3} X_{2}(1)+\frac{1}{\tau+3} X_{3}(1)+\frac{1}{\tau+3} X_{4}(1)$. Similarly, other team members will state $\frac{1}{\tau+1} X_{1}(1)+\frac{\tau}{\tau+1} X_{i}(1)$ in the second round $(i=2,3,4)$. In rounds 3 though 6 , however, in contrast to the 'Standard' model, the other team members will not copy the team

[^7]leader's second-round answer. Instead, driven by overconfidence, team member $i$ will state $\frac{\tau s_{i}+\sum_{j \neq i} s_{j}}{\tau s_{i}+\sum_{j \neq i} s_{j}+\tau f_{i}+\sum_{j \neq i} f_{j}}$, which can be rewritten as $\frac{\tau}{\tau+3} X_{i}(1)+\sum_{j \neq i} \frac{1}{\tau+3} X_{j}(1)$.

Overall, the updating in the 'Standard Plus' model can be summarized as follows:
Summary 3 (Standard-Plus Model). The team leader computes a weighted average of all team members' first-round answers and states $\frac{\tau}{\tau+3} X_{1}(1)+\frac{1}{\tau+3} X_{2}(1)+\ldots+\frac{1}{\tau+3} X_{4}(1)$ from round 2 on, other team member $i$ states $\frac{1}{\tau+1} X_{1}(1)+\frac{\tau}{\tau+1} X_{i}(1)$ in round $2(i=2,3,4)$, while stating $\frac{\tau}{\tau+3} X_{i}(1)+\sum_{j \neq i} \frac{1}{\tau+3} X_{j}(1)$ from round 3 on.

## The 'Sophisticated Plus' model

By the 'Sophisticated Plus' model, we denote the extension of the 'Sophisticated' model by overprecision. The only difference to the 'Sophisticated' model is that we assume that team members misinterpret their signal: as above, when the signal actually is $\psi_{i}=\left(s_{i}, f_{i}\right)$, team member $i$ will interpret it as if the received signal was $\left(\tau s_{i}, \tau f_{i}\right)$. Therefore, team member $i$ 's belief in the first round will be given by a $\operatorname{Beta}\left(1+\tau s_{i}, 1+\tau f_{i}\right)$ distribution, leading to the first-round answer $\frac{\tau s_{i}}{\tau s_{i}+\tau f_{i}}=\frac{s_{i}}{n_{i}}$, as in the 'Sophisticated' model. However, after learning from the other team members, the second-round answers are still biased by overprecision: from round 2 on, the team leader will state $\frac{\tau s_{1}+s_{2}+\ldots+s_{4}}{\tau s_{1}+s_{n}+\ldots+s_{4}+\tau f_{1}+f_{2}+\ldots+f_{4}}$, which can be rewritten as $\frac{\tau n_{1}}{\tau n_{1}+n_{2}+n_{3}+n_{4}} X_{1}(1)+\frac{n_{2}}{\tau n_{1}+n_{2}+n_{3}+n_{4}} X_{2}(1)+\frac{n_{3}}{\tau n_{1}+n_{2}+n_{3}+n_{4}} X_{3}(1)+\frac{n_{4}}{\tau n_{1}+n_{2}+n_{3}+n_{4}} X_{4}(1)$. Similarly, other team members will state $\frac{n_{1}}{n_{1}+\tau n_{i}} X_{1}(1)+\frac{\tau n_{i}^{2}}{n_{1}+\tau n_{i}} X_{i}(1)$ in the second round $(i=2,3,4)$. In rounds 3 though 6, however, in contrast to the Sophisticated Model, the other team members will not copy the team leader's second-round answer. Instead, driven by overconfidence, team member $i$ will state $\frac{\tau s_{i}+\sum_{j \neq i} s_{j}}{\tau s_{i}+\sum_{j \neq i} s_{j}+\tau f_{i}+\sum_{j \neq i} f_{j}}$, which can be rewritten as $\frac{\tau n_{i}}{\tau n_{i}+\sum_{j \neq i} n_{j}} X_{i}(1)+\sum_{j \neq i} \frac{n_{j}}{\tau n_{i}+\sum_{j \neq i} n_{j}} X_{j}(1)$.

Overall, the updating in the 'Sophisticated Plus' model can be summarized as follows:
Summary 4 (Sophisticated-Plus Model). The team leader computes a weighted average of all team members' first-round answers and states

$$
\frac{\tau N_{1}}{\tau N_{1}+N_{2}+N_{3}+N_{4}} X_{1}(1)+\sum_{j=2}^{4} \frac{N_{j}}{\tau N_{1}+N_{2}+N_{3}+N_{4}} X_{j}(1)
$$

from round 2 on, other team member $i$ states $\frac{N_{1}}{N_{1}+\tau N_{i}} X_{1}(1)+\frac{\tau N_{i}}{N_{1}+\tau N_{i}} X_{i}(1)$ in round $2(i=2,3,4)$, while stating

$$
\frac{\tau N_{i}}{\tau N_{i}+\sum_{j \neq i} N_{j}} X_{i}(1)+\sum_{j \neq i} \frac{N_{j}}{\tau N_{i}+\sum_{j \neq i} N_{j}} X_{j}(1)
$$

from round 3 on.

## B. 5 Specification and Extension of Naïve Models

## B.5.1 Specific DeGroot Models

In the DeGroot framework of naïve learning, agents approach consensus. Consensus is given by $x(\infty)=w^{\prime} x(1)$, where the vector $w$ captures the eigenvector centrality of the agents (e.g., Friedkin, 1991; DeMarzo et al., 2003; Golub and Jackson, 2010).

The most common specification is to allocate equal weights to any connection including to oneself.

$$
G=\left(\begin{array}{cccc}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 \\
\frac{1}{2} & 0 & 0 & \frac{1}{2}
\end{array}\right)
$$

Credit for this specification is usually given to DeMarzo et al. (2003). This behavior corresponds to Bayesian updating with independent signals of equal precision in the first round, but not in later rounds. The long-term prediction using this DeMarzo et al. Model is determined by $w=\left(\frac{2}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)^{\prime}$, i.e., pendants' initial opinions enter the calculation of the consensus with a weight of $20 \%$ each, while the center's initial opinion accounts for $40 \%$ of the consensus.

Corazzini et al. (2012) suggest improving the DeMarzo et al. Model by increasing the weight of agents who listen to many other agents (and show that this twist improves the model fit to experimental data). The suggested specification is that the weights are proportional to the outdegree (i.e., the number of agents listened to):

$$
G=\left(\begin{array}{cccc}
\frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{3}{4} & \frac{1}{4} & 0 & 0 \\
\frac{3}{4} & 0 & \frac{1}{4} & 0 \\
\frac{3}{4} & 0 & 0 & \frac{1}{4}
\end{array}\right)
$$

This model predicts that the center of the star is even more influential in the long run: $w=$ $\left(\frac{9}{15}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}\right)^{\prime} \cdot{ }^{14}$

## B.5.2 Models of Naïve Learning with Conservatism

Incorporating conservatism requires a model extension. Friedkin and Johnsen (1990) provide a more general model of naïve learning. Initial opinions are determined by some exogenous conditions, which can always have an impact on an agent's opinion. Such a model has also been analyzed in Golub and Jackson (2012). To incorporate this aspect, we can simply let agents stick to their initial guess $x_{i}(1)$ to some extent $\alpha$ :

$$
x_{i}(t)=\left(1-\alpha_{i}\right) \cdot e_{i}^{\prime} G x(t-1)+\alpha_{i} \cdot x_{i}(1) .
$$

[^8]For $\alpha_{i}=0$, we have the DeGroot model. For $\alpha_{i}=1$, we have the simplest conceivable model: an agent makes an initial guess $x_{i}(1)$ and then sticks to it. This is a baseline model that we call the Sticking Model, as already mentioned when discussing totally overprecise rational learners.

If $\alpha_{i} \in(0,1)$ for every agent $i$, then the model prediction is that agents move towards the others' guesses, but still rely on their initial guess. This is conservatism. ${ }^{15}$ Interestingly, with this model variation, the updating process converges without reaching a consensus (for generic starting values)!

We extend the DeMarzo et al. Model and the Corazzini et al. Model by the Friedkin and Johnsen (1990) framework and set the conservatism parameter $\alpha=0.5$. This leads to the DeMarzo et al. Plus Model and the Corazzini et al. Plus Model. In these models, agents do not approach consensus anymore. For instance, in the DeMarzo et al. Plus Model, the longterm guess of a pendant $i$ is a convex combination of initial guesses with the following weights: weight $\frac{2}{9}$ on the center's initial guess, weight $\frac{1}{27}$ on other pendants' initial guesses each, and weight $\frac{19}{27}(\approx 70 \%)$ on the own initial guess, which leads to different guesses of each pendant. This difference is illustrated in the right panels of Figure B.1.

Four models are illustrated in Figure B.1. In this example, initial answers are $x_{1}=20 \%$ for the center, and $x_{2}=40 \%, x_{3}=60 \%$, and $x_{4}=80 \%$ for the pendants. The most important differences are easily observable. In Bayesian models (left panels), learning stops in round 3 ; in naïve models (right panels), answers converge over time. In the specifications without conservatism (upper panels), agents reach or converge to consensus; in the models with conservatism (lower panels), there is a persistent heterogeneity of answers, such that each agent's answer is "biased" towards the own initial answer. Note that the conservative agents in the naïve models behave similarly to conservative agents in the rational learning approach.

[^9]

Figure B.1: Examples of dynamics with time on the abscissa and answers (in percentage points) on the ordinate. Upper panels illustrate two prominent models from the literature; lower panels illustrate their extensions when conservatism is incorporated. Standard Model is upper left, Standard-Plus Model is lower left, DeMarzo et al. Model is upper right, and DeMarzo et al. Plus Model is lower right panel. Hence the left panels illustrate rational models, the right panels naïve models.

## C Appendix: Details of the Experimental Design

The experiment was run in eleven sessions (which followed after two pilot sessions) in August and September 2013. It was conducted in the experimental laboratory of the Faculty of Economic and Social Sciences at the University of Hamburg, Germany. It was programmed using zTree (Fischbacher, 2007) and organized and recruited with hroot (Bock et al., 2014). In total, 176 university students with various academic backgrounds participated in the experiment (no exclusions to the pool applied). The participants earned on average EUR 9.50. The norm at the lab was EUR 10. Each session lasted approximately 60 minutes, including instructions, questionnaire and payments.

Subjects were randomly assigned to computer terminals. After all participants were seated, two sets of instructions were handed out, a German version as well as an English translation. The German instructions were read aloud to establish common knowledge. The subjects were then given the possibility to ask questions, which were answered privately. The instructions were left with the participants for reference during the whole experiment. In the instructions, it was pointed out that the use of mobile phones, smart phones as well as tablets or similar devices would lead to expulsion from the experiment and exclusion from all payments. ${ }^{16}$ There were no data exclusions.

All decisions and the payments at the end of the experiment were made anonymously. The participants were not informed about the identity of any other participant and they were paid privately upon completion of each session. The individual computer terminals were separated by boards and could be partially closed with curtains.

## C. 1 Experimental Task

The design of the experiment draws upon the studies by Lorenz et al. (2011), Rauhut and Lorenz (2011), and Moussaïd et al. (2013). The subjects were asked to give estimates on factual questions and to state their confidence level. The experiment was based on questions with hard facts, because they admit an unambiguously correct answer. For instance, voter turnout in a specific election is officially counted and reported. The questions for the experiment were chosen from a pool of questions that were used in previous studies, in particular in the three studies just cited above. The questions cover various fields of knowledge. The questions were chosen so that subjects were unlikely to know the exact answer. At the same time, questions for which they did not have any knowledge at all were avoided. In order to avoid highly skewed responses, the questions were such that the correct answers lay in an interval of $0 \%$ to $100 \%$. The complete list of questions is reported in Table C.1. Participants could answer with any integer number between (and including) 0 and 100. The time to answer a question was not limited, the subjects were, however, given a reference time of 25 seconds per answer. The remaining time could be observed on the screens, but participants were informed beforehand that running out of time did not bear any consequences.

[^10]| Phase | Ident. | Question | Correct Answer |
| :---: | :---: | :--- | :---: |
|  | A1 | What was the voter turnout of the federal elections in Germany in 2005? | 78 |
|  | A2 | What is the share of water in a cucumber? | 95 |
|  | A3 | What share of the world-wide land area is used for agriculture? | 18 |
| I | A4 | What is the percentage of the world's population that lives in North- and Southamerica? | 14 |
|  | A5 | What is the percentage of the world's population between 15 and 64 years old? | 65 |
|  | A6 | What is the percentage of female professors in Germany? | 18 |
|  | A7 | What is the share of people with blood type B (BB or B0)? | 11 |
|  | A8 | What is the percentage of the world's roads (paved and unpaved) that are in India? | 11 |
|  |  |  |  |
|  | B1 | What was the voter turnout of the federal elections in Germany in 2009? | 71 |
|  | B2 | What is the share of water in an onion? | 89 |
|  | B3 | What share of the working population is working in the agricultural sector? | 40 |
| II | B4 | What is the percentage of the world's population that lives in Africa? | 15 |
|  | B5 | What is the percentage of the world's population older than 15, that can read and write? | 82 |
|  | B6 | What is the percentage of female Nobel laureates in literature (until 2010)? | 11 |
|  | B7 | What is the share of people with blood type A (AA or A0)? | 43 |
|  | B8 | What is the percentage of the world's airports that are located in the United States? | 30 |

Table C.1: Overview of all Questions

Confidence was measured on a nine point scale from 0 to $65+$. Each value indicated a range of expected deviation of the individual estimate from the true value. The scale was explained in the instructions. For better understanding, a verbal interpretation was added. Table C. 2 appeared in the instructions and gives a detailed description. As it can be seen in Table C.2, the distances between successive items are increasing. A simple nine point scale from 1 to 9 would have created the impression of equivalence of the distances. To avoid misinterpretation of the scale, the values on the scale used in the experiment directly corresponded to the expected range of deviations. The same values were displayed on the screens. The confidence indication was not (directly) incentivized.

## C. 2 Phase I

The experiment consisted of two phases. The first set of instructions was handed out to the participants before the first phase. In phase I, each subject had to answer eight questions and indicate his confidence level. Each question was answered once. An English translation to the questions was provided on the screens. The order of the questions was randomized over the participants. The subjects were informed that there was going to be a second phase with new instructions and that their choices in phase I might affect phase II. However, participants did not know the relation between phase I and phase II. In particular, they were given the instructions for phase II only after phase I was finished. Figure C. 1 shows a screen shot of phase I.

Table C.2: Summary of the Confidence Scale and its Interpretation
Scale I assume that my estimation most likely (in nine Concerning my estimation I am of ten cases) does not deviate by more than

| 0 | 0 percentage points from the true value | absolutely confident |
| :---: | :--- | :--- |
| 1 | 1 percentage points from the true value | pronouncedly confident |
| 2 | 2 percentage points from the true value | very confident |
| 4 | 4 percentage points from the true value | rather confident |
| 8 | 8 percentage points from the true value | partially confident |
| 16 | 16 percentage points from the true value | rather unconfident |
| 32 | 32 percentage points from the true value | very unconfident |
| 64 | 64 percentage points from the true value | pronouncedly unconfident |
| $65+$ | 65 percentage points or more from the true value | absolutely unconfident |

## C. 3 Phase II

Before phase II started, a new set of instructions was handed out and read aloud. The participants kept the first set of instructions. After participants were offered to ask questions, which were again answered privately, phase II started. The participants were randomly matched into groups of four. These groups stayed fixed. Four is the minimum number of individuals required for a star network that is no simple line. The groups were fixed for the remainder of the experiment. The subjects were again asked to answer eight questions, but in phase II each question was answered six times ( $t=1, \ldots, 6$ estimation periods). In the first estimation period, the subjects individually answered the questions and stated their confidence level. The first period was, therefore, analogous to phase I. In the second estimation period, the subjects were informed about the other group members' guesses in period one according to their position in a star network. The subject at the central node could observe all answers and confidence information given by the members of her group. The pendants could observe the answer and confidence information given by their group's center. In addition, everyone was shown their own last answer and confidence statement. Participants then submitted new estimates and confidence levels. This procedure was then repeated four times, with the information from the respective previous round. It was communicated in the instructions for both phases that the payment, although they were playing in groups, was based solely on the individual error. The order of the questions was randomized across groups. The subjects only had to wait for their group members for each estimation period, but had to wait for all participants of the same session to enter a new question round. Figures C. 2 and C. 3 show screen shots of phase II.

## C. 4 Treatments

The selection criterion for the individual at the center of the star network varied with the treatments. The center was fixed for one question round, that is for six estimation periods.


Figure C.1: Screen Shot of phase I

The pendants were named $A, B$, and $C$. The names changed with each question round. The positions and the selection criteria were communicated on screen at the beginning of each question round. The criterion changed once after the fourth question round. This piece of information was communicated to the subjects in the instructions for phase II.

The selection was either random or it was based on information from phase I. Each question in phase II had a partner question in phase I. The partner questions were two questions considered similar to each other and from the same field of interest. It could be expected that the individual error and confidence levels for the two partner questions were correlated. It was communicated that questions in phase II partially resembled questions from phase I, but were never identical.

Treatment T1 was a high accuracy treatment. This type of selection rule is based on the quality of the estimates of the partner questions in phase I. In each group, the agent who had made the smallest error in her estimation of the answer to the partner question was chosen to be in the center for this question round. Treatment T2 was a high confidence treatment. The selection in treatment T2 was based on the confidence level indicated in phase I for the partner question. The agent with the highest confidence within each group was selected. Treatment T0 was a control treatment. In this treatment, the central agent was determined by a random pick. It was also made clear that in case of a tie in T1 or T2, a random choice was made between the group members who were eligible for the center position.

It was necessary to separate the question determining the central node from the actual question of interest and, therefore, include two phases in the experiment for several reasons. If the selection had been based on the first estimation period, by communicating that the center had had the best initial guess on that question in treatment T1, the participants would have gained outside information about the true state of the world. With the introduction of the partner question, it was possible to reduce the level of outside information to a minimum.


Figure C.2: Screen Shot of phase II from the Viewpoint of a Pendant

Furthermore, the questions were asked in successive rounds, i.e., the second to sixth period of every question round immediately followed the initial period. Communication of the selection mechanism could have induced strategic considerations on the side of the participants. Particularly in treatment T2, a misrepresentation of the confidence level to get the desired position in the network could have been expected. In order to rule out intentional misrepresentation, we collected the data in phase I before its role in the determination of the network of phase II was communicated.

## C. 5 Permutation of Treatments

To avoid session effects, each treatment was run at each session. Each group played two different treatments in a fixed order. They started with either treatment T1 or T2 and then changed to treatment T 0 after the fourth question round, or they started with treatment T 0 and then changed to treatment T 1 or T2. Table C. 3 shows for all sessions which group played which question in which order and in which treatment. Each line stands for one group. The first column is the group number, the last column the session. For instance, in session 1 there were four groups labeled 1 to 4 . Group 1 first played a set of eight questions in phase I in random order. Then, in phase II, they played eight partner questions in the same order, the first four of them in the accuracy treatment T 1 and the remaining four in the random treatment T 0 .

We have 44 groups (consisting of 4 subjects each) who all played 8 different questions. Each group played 4 questions in one condition (random treatment T0) and 4 questions in another (either accuracy treatment T1 or confidence treatment T2). Table C. 4 shows a detailed breakdown of the distribution of participants over the treatments.

This makes 352 unique group-question pairs, of which 176 are in the random treatment T0,


Table C.3: Overview of all sessions with question order. In phase II black is the random treatment T 0 , gray is the accuracy treatment T 1 , and light gray is the confidence treatment T2.

| Bitte beantworten Sie die folgende Frage. <br> Wie viel Prozent aller Erwerbstätigen der Welt arbeiten im Landwirtschaftssektor? What share of the working population is working in the agricultural sector? <br> Bitte nur ganze Zahlen zwischen 0 und 100 eingeben. <br> Das Prozentzeichen bitte nicht eingeben. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Schäzung | Vertrauensangabe |
|  |  | Inre Angaben | 35 | 8 |
| Sie sind Zentrumsspieler. |  | Außenspieler A | 23 | $65+$ |
|  |  | Außenspieler B | 5 | 16 |
|  |  | Außenspieler C | 21 | 64 |
| Ihre Antwort: 1 |  |  |  |  |
| Wie sicher sind Sie sich bei Ihrer Antwort? <br> absolut sicher $\subset \subset \subset C \subset C \subset C C$ absolut unsicher |  |  |  |  |

Figure C.3: Screen Shot of phase II from the Viewpoint of a Center

Table C.4: Summary of Treatments

| Treatment | Participants |
| :--- | :---: |
| T0 Random, T1 Accuracy | 44 |
| T0 Random, T2 Confidence | 40 |
| T1 Accuracy, T0 Random | 44 |
| T2 Confidence, T0 Random | 48 |

88 in the accuracy treatment T 1 , and 88 in the confidence treatment T 2 , as summarized by Table 1. (Since one group-question pair consists of four people who answer six times the same question in phase II, this yields 8,448 single decisions in phase II.)

## C. 6 Payment

The subject's payment was based on the individual error of the estimation. The error was calculated as the absolute difference between the estimation and the correct answer. The individual error was converted into game points as described in Table C.5.

One question was randomly selected for payment in phase I. In phase II, one estimation period was randomly chosen for each question round. The choice was identical for everyone. The monetary incentive in this form encouraged the subjects to find the true answers. Payoffs only depended on one's own decisions. There is neither an incentive to improve others' choices

Table C.5: Summary of Payments

| Distance | Points |
| :--- | :---: |
| 0 percentage points | 16 game points |
| 1 percentage point | 8 game points |
| 2 percentage points | 4 game points |
| 3 or 4 percentage points | 2 game points |
| $5,6,7$ or 8 percentage points | 1 game point |
| more than 9 percentage points | 0 game points |

nor to be better than others. The experimental design put subjects into a position in which they would try to get as close to the truth as possible by using their own knowledge and information from others (Lorenz et al., 2011). The game points were converted with an exchange rate of EUR 0.3 per point. The total payment in EUR was:

5 show up fee + game points from phase I $\cdot 0.3+$ game points from phase II $\cdot 0.3$.
The maximum payment possible was EUR 48.2. The experiment was concluded by a short questionnaire. After all participants had finished answering the questionnaire, the correct solutions to all estimation questions were displayed on screen. The participants were then paid anonymously at two cash desks at the exits of the laboratory.

## C. 7 Pretest Sessions

Two pretest sessions were run on the 6th and 19th of August, 2013 with 15 and 16 participants, respectively, in order to calibrate the number of questions and the conversion rate. We determined the sample size prior to the experiment and the pretest sessions by a heuristic statistical power analysis.

## D Instructions

The original instructions are written in English and in German. On the next pages, we provide the original instructions, first for phase I of the experiment, then for phase II.

## Herzlich Willkommen zum Experiment!

Sie nehmen nun an einem Experiment zur ökonomischen Entscheidungsfindung teil. Bitte beachten Sie, dass ab nun und während des gesamten Experiments keine Kommunikation gestattet ist. Wenn Sie eine Frage haben, strecken Sie bitte die Hand aus der Kabine, einer der Experimentatoren kommt dann zu Ihnen. Während des gesamten Experiments ist das Benutzen von Handys, Smartphones, Tablets oder Ähnlichem untersagt. Bitte beachten Sie, dass eine Zuwiderhandlung zum Ausschluss von dem Experiment und von sämtlichen Zahlungen führt.

Sämtliche Entscheidungen erfolgen anonym, d.h. keiner der anderen Teilnehmenden erfährt die Identität des Anderen. Auch die Auszahlung erfolgt anonym am Ende des Experiments. Das bedeutet, dass keiner der anderen Teilnehmenden erfährt, wie hoch Ihre Auszahlung ist.

## Anleitung zum Experiment und allgemeine Informationen

Das Experiment besteht aus zwei Phasen. Sie erhalten zunächst die Instruktionen für die Phase I des Experiments. Die Instruktionen für die Phase II erhalten Sie nachdem alle Teilnehmenden die erste Phase abgeschlossen haben. Ihre Angaben in Phase I können in manchen Fällen Einfluss auf Phase II haben. Auf Phase II folgt ein kurzer Fragebogen.

## Informationen zu Phase I des Experiments

In diesem Experiment geht es um das möglichst gute Einschätzen von bestimmten Größen. Je nach Qualität Ihrer Schätzungen erhalten Sie Punkte, die am Ende des Experiments zu Ihrer Auszahlung in Euro führen.

In der ersten Phase des Experiments werden Sie gebeten, acht Fragen zu beantworten. Gefragt ist jeweils nach einem Prozentwert und Sie können stets nur ganze Zahlen zwischen (und einschließlich) 0 und 100 als Schätzung angeben. Das Prozentzeichen soll dabei nicht eingegeben werden. Die wahren Werte beruhen auf offiziellen Statistiken und wurden, insofern dies nötig war, auf ganze Zahlen gerundet.

Sie werden außerdem gebeten, Ihr Vertrauen in Ihre Schätzung auf einer Skala anzugeben (Vertrauensangabe). Bitte entnehmen Sie die Bedeutung der Werte auf der Skala der folgenden Übersicht, die jedem Zahlenwert auch eine verbale Interpretation beifügt:

| Skala | Ich gehe davon aus, dass meine Schätzung <br> höchstwahrscheinlich $^{*}$ nicht mehr als | Ich bin mir bei meiner Schätzung: |
| :--- | :--- | :--- |
| 0 | 0 Prozentpunkte vom wahren Wert abweicht. | absolut sicher |
| 1 | 1 Prozentpunkt vom wahren Wert abweicht. | ausgesprochen sicher |
| 2 | 2 Prozentpunkte vom wahren Wert abweicht. | sehr sicher |
| 4 | 4 Prozentpunkte vom wahren Wert abweicht. | eher sicher |
| 8 | 8 Prozentpunkte vom wahren Wert abweicht. | teilweise sicher |
| 16 | 16 Prozentpunkte vom wahren Wert abweicht. | eher unsicher |
| 32 | 32 Prozentpunkte vom wahren Wert abweicht. | sehr unsicher |
| 64 | 64 Prozentpunkte vom wahren Wert abweicht. | ausgesprochen unsicher |
| $65+$ | 65 Prozentpunkte oder mehr vom wahren Wert <br> abweicht. | absolut unsicher |

## Beispiel:

Nehmen wir an, Ihre Schätzung beträgt $50 \%$ und Sie sind sich bei dieser Schätzung „eher sicher" (Skalenwert 4). Das bedeutet, dass Sie davon ausgehen, dass Ihre Schätzung von 50\% höchstwahrscheinlich (in 9 von 10 Fällen) nicht mehr als 4 Prozentpunkte von dem wahren Wert abweicht, der wahre Wert also zwischen 46\% und 54\% liegt.

Grafik 1 zeigt beispielhaft, welche Bildschirmoberfläche Sie bei jeder Frage erwartet. In das Eingabefeld für die Schätzung, soll eine Zahl zwischen 0 und 100 eingegeben werden. Darunter erfolgt die Angabe des Vertrauens auf der angezeigten Skala. Bitte bestätigen Sie Ihre Eingaben durch Klick auf den Weiter-Button (nicht ersichtlich in Grafik 1).


## Berechnung Ihres Einkommens aus Phase I

Grundlage für die Gewinnberechnung ist der Abstand Ihrer Schätzung zum richtigen Wert - je näher Sie am richtigen Wert liegen, desto mehr Geld erhalten Sie. Der Abstand wird berechnet als der absolute Betrag der Differenz zwischen Ihrer eigenen Schätzung und dem wahren Wert. Ihr Gewinn hängt ausschließlich von Ihrer eigenen Schätzung ab.

## Punktevergabe:

- 16 Punkte erhalten Sie, wenn Ihre Schätzung exakt den richtigen Wert trifft (0 Prozentpunkte Abstand).
- 8 Punkte erhalten Sie, wenn Ihre Schätzung fast exakt den richtigen Wert trifft (1 Prozentpunkt Abstand).
- 4 Punkte erhalten Sie für eine kleine Abweichung der Schätzung vom wahren Wert (2 Prozentpunkte Abstand).
- 2 Punkte erhalten Sie für eine mittlere Abweichung der Schätzung vom wahren Wert (3 oder 4 Prozentpunkte Abstand).
- 1 Punkt erhalten Sie für eine größere Abweichung der Schätzung vom wahren Wert (5, 6, 7, oder 8 Prozentpunkte Abstand).
- Weicht Ihre Schätzung stark vom richtigen Wert ab (Abstand von 9 Prozentpunkten und mehr), erhalten Sie für diese Runde keine Punkte.

Für Ihr Einkommen in Phase I wird aus den 8 Fragen zufällig eine auszahlungsrelevante Frage ausgelost. Ihr Einkommen ergibt sich dann durch Ihre dort erzielte Punktzahl, wobei folgender Wechselkurs gilt: 1 Punkt entspricht $\mathbf{0 , 3 0} €$. Das maximal mögliche Einkommen in Phase I beträgt 4,80 €.

## Beispiel (fortgesetzt):

Sie haben bei einer Frage 50\% geschätzt. Nehmen wir an, dass der wahre Wert bei $48 \%$ liegt, dann beträgt Ihr Abstand 2. Wenn diese Frage als auszahlungsrelevant ausgelost wird, dann bekommen Sie 4 Punkte und damit 1,20 € ausgezahlt.

## Gesamteinkommen

Ihr Gesamteinkommen aus dem Experiment setzt sich aus den garantierten $5 €$, plus Ihrem Einkommen aus Phase I, plus Ihrem Einkommen aus Phase II zusammen und wird am Ende des Experiments ausgezahlt.

Viel Erfolg!

## Welcome to today's experiment!

You are now participating in an experiment concerning economic decision making. Please note that from now on and during the time of the experiment communication is not allowed. If you have any questions, please indicate this by showing your hand outside of the individual cabin; one of the experimenters will come to assist you. The use of mobile phones, tablet PCs and similar devices is not allowed during the time of the experiment. Please note that a violation of this rule will lead to an expulsion of the experiment and will exclude you from any payment.

All decisions are made anonymously, i.e. none of the other participants will get to know the identity of a decision maker. Similarly, the payment is made anonymously such that none of the other participants will get to know how much you earn.

## Instructions and general information

The experiment consists of two phases. You are now holding the instructions for phase I. You will receive the instructions for phase II after all participants have completed phase I. In some cases the choices in phase I might affect phase II. After phase II there will be a short questionnaire to answer.

## Information about phase I of the experiment

This experiment is about estimating certain figures as accurately as possible. Your score depends on the quality of your estimations and will be transformed into a payment in Euros at the end of the experiment.

In the first phase of the experiment you are asked to answer eight questions. Each questions is about some percentage of a face value and you can type in integer numbers between (and including) 0 and 100 as an estimate. Thereby, the percent sign should not be typed in. The true values are based on some official statistical reports and were, if applicable, rounded to the next integer.

In addition, we ask you for your confidence in your estimate on a scale. The meaning of each value of the scale can be found in the following table, which adds a verbal interpretation to each quantity.

| Scale | I assume that my estimation most likely* <br> does not deviate by more than | Concerning my estimation I am |
| :--- | :--- | :--- |
| 0 | 0 percentage points from the true value. | absolutely confident |
| 1 | 1 percentage points from the true value. | pronouncedly confident |
| 2 | 2 percentage points from the true value. | very confident |
| 4 | 4 percentage points from the true value. | rather confident |
| 8 | 8 percentage points from the true value. | partially confident |
| 16 | 16 percentage points from the true value. | rather unconfident |
| 32 | 32 percentage points from the true value. | very unconfident |
| 64 | 64 percentage points from the true value. | pronouncedly unconfident |
| $65+$ | 65 percentage points or more from the true <br> value. | absolutely unconfident |

(* in 9 out of 10 cases)

## Example:

Suppose your estimation is $50 \%$ and concerning this estimation you feel „rather confident" (value 4 on scale). This means that you assume that your estimation most likely (in 9 out of 10 cases) does not deviate by more than 4 percentage points from the true value, i.e. that the true value lies within $46 \%$ and $54 \%$.

Figure 1 gives an example for the screen which you will see for each question. The first input is your estimation, which must be a number between 0 and 100. The second input is your confidence level. Please confirm your choices by clicking on the „Weiter" button (which is not illustrated in Figure 1).


## Calculation of your income from phase I

Profits are based on the distance of your estimation to the true value - the closer you are to the true value, the more money you earn. The distance is computed as the absolute value of the difference between your estimation and the true value. Your profit solely depends on your own estimation.

## Score:

- You receive $\mathbf{1 6}$ points if your estimation hits exactly the true value (distance of $\mathbf{0}$ percentage points).
- You receive $\mathbf{8}$ points if your estimation hits almost exactly the true value (distance of $\mathbf{1}$ percentage point).
- You receive 4 points for a small distance of your estimation to the true value (distance of $\mathbf{2}$ percentage points).
- You receive $\mathbf{2}$ points for a medium distance of your estimation to the true value (distance of 3 or 4 percentage points).
- You receive $\mathbf{1}$ point for a larger distance of your estimation to the true value (distance of 5, 6, 7, or 8 percentage points).
- If your estimation strongly deviates from the true value (distance of 9 percentage points or more), then you receive no points in this round.

To generate your income of phase I, one of the 8 questions will be randomly drawn to be payoffrelevant. Your income in phase $I$ is then derived from your score in this question, whereas the following exchange rate applies: $\mathbf{1}$ point corresponds to $\mathbf{0 , 3 0} €$. The maximal possible income in phase I is $4,80 €$.

## Example (continued):

You have estimated $50 \%$ in some question. Let us suppose that the true value is $48 \%$. Then your distance is 2 . If this question is drawn to be payoff-relevant, then you receive 4 points such that you will earn 1,20 €.

## Total income

Your total income from the experiment consists of the guaranteed $5 €$, plus your income from phase $I$, plus your income from phase II, and will be paid by the end of the experiment.

Good luck!

## Informationen zu Phase II des Experiments:

In Phase II werden Sie erneut gebeten, 8 unterschiedliche Fragen zu beantworten und Ihre jeweilige Vertrauensangabe zu machen. Bitte beachten Sie, dass die Fragen in Phase II den Fragen aus Phase I des Experiments teilweise ähnlich sind, sie sind jedoch in keinem Fall identisch! Nach Ihrer ersten Schätzung zu einer Frage werden Sie noch fünf weitere Male gebeten, eine Schätzung für die gleiche Frage abzugeben. Ab der ersten Wiederholung bekommen Sie je nach Spielmodus Informationen über die Angaben anderer Spieler.

Zu Beginn der Phase II werden Sie entweder in eine Vierergruppe eingeteilt oder Sie spielen diese Phase einzeln. Sowohl die Zuordnung als auch die Zusammensetzung der Gruppen erfolgen zufällig und ändern sich während des Experiments nicht mehr.

## Gruppenmodus

In jeder Vierergruppe wird ein Spieler für die Dauer einer Frage für die Rolle des Zentrumsspielers ausgewählt, während die drei anderen die Außenspieler sind (Grafik 2). Der Auswahlmechanismus wird jeweils bekannt gegeben und wechselt einmal nach der vierten Frage. Die Auswahl basiert entweder auf Angaben aus Phase I oder erfolgt zufällig.


Sie werden nun sechs Mal geben, eine Schätzung für die gleiche Frage abzugeben. In der ersten Schätzrunde stehen noch keine Informationen zur Verfügung. Von der zweiten bis zur sechsten Schätzrunde sieht der Zentrumsspieler die vorangegangenen Schätzungen und Vertrauensangaben der Außenspieler, während die Außenspieler die Schätzung und Vertrauensangabe des Zentrumsspielers sehen, nicht jedoch die Eingaben der jeweils anderen Außenspieler.

Grafiken 3 und 4 zeigen beispielhaft, wie die Bildschirmoberflächen für einen Außenspieler und einen Zentrumsspieler aussehen können.



Bitte beachten Sie, dass die Bezeichnungen A, B und C für jede Frage, also für sechs Antworten, bestehen bleiben und dann neu vergeben werden.

Im Einzelmodus werden ebenfalls 6 Schätzungen zu jeder Frage abgegeben. Die dabei bereitstehenden Informationen werden für jede Frage auf dem Bildschirm erläutert. Die Art der Information wechselt einmal nach der vierten Frage.

## Berechnung Ihres Einkommens aus Phase II

Für die Berechnung des Einkommens aus Phase II wird für jede der 8 Fragen genau eine auszahlungsrelevante Runde zufällig durch den Computer bestimmt. Genau wie in Phase I ist der Abstand Ihrer Schätzung zum wahren Wert Grundlage für die Gewinnberechnung. Je näher Sie in der zufällig ausgewählten Runde am richtigen Wert liegen, desto mehr Geld erhalten Sie (siehe Punktevergabe in den Instruktionen zu Phase I). Bitte beachten Sie, dass, auch wenn Sie in einer Gruppe spielen, nur Ihre eigene Schätzung Einfluss auf Ihren Gewinn hat.

Ihr Einkommen aus Phase II ergibt sich aus der Summe der Punkte, die Sie für jede Frage in der jeweils auszahlungsrelevanten Runde gesammelt haben, wobei nach wie vor der Wechselkurs von 1 Punkt entspricht 0,30 € gilt. Das maximal mögliche Einkommen in Phase II beträgt 38,40 €.

## Gesamteinkommen

Ihr Gesamteinkommen aus dem Experiment setzt sich aus den garantierten $5 €$, plus Ihrem Einkommen aus Phase I, plus Ihrem Einkommen aus Phase II zusammen.

Ihre Auszahlung sowie die tatsächlichen Werte erfahren Sie am Ende des Experiments.

## Viel Erfolg

## Information concerning phase II of the experiment

In phase II you are asked again to answer 8 questions and to provide your confidence levels. Please note that the questions of phase II partially resemble the questions of phase I, but they are never identical! After your first estimation concerning one question you will be asked 5 further times to provide an estimation for the same question. After the first repetition - depending on the mode of play - you will receive information about other players' decisions.

At the beginning of phase II you are either assigned into a group of four players or you will be a single player. Both the assignment and the composition of the groups are generated randomly and will not change for the time of the experiment.

## Group mode

In each group of four, one player is selected to be the central player for the time of one question, while the other three are peripheral players (Figure 2). The selection mechanism is announced each time and will once change after four questions. The selection is either based on inputs from phase I or is made randomly.


You will then be asked 6 times to provide an estimation for the same question. In the first round of estimation no information is provided. From the second round up to the $6^{\text {th }}$ round the central player can see the previous estimations and confidence choices of the peripheral players, while the peripheral players can see the previous estimation and confidence choice by the central player, but not those of the other peripheral players.

Figure 3 and 4 illustrate how computer screens of a peripheral player and of a central player might look like.


| Bitte beantworten Sie die folgende Frage. <br> Wie viel Prozent der Erdoberflache ist von Wasser bedeckt? What percentage of the earth's surface is covered by water? <br> Bitte nur ganze Zahlen zwischen 0 und 100 eingeben. <br> Das Prozentzeichen bitte nicht eingeben. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Schatung | Vetrrauensangabe |
|  |  | Inre Angaben | 43 | 32 |
| Sie sind Zentrumsspieler. |  | AuBenspieler A | ${ }^{23}$ | ${ }^{64}$ |
|  |  | AuBenspieter B | 0 | 0 |
|  |  | AuBenspieler C | 0 | 0 |
| Ihre Antwort: $\square$ |  |  |  |  |
|  |  |  |  |  |

Please note that the labels $A, B$, and $C$ are fixed for each question, i.e. for six answers, and then they are newly assigned.

In the single-player mode you also have to provide 6 estimations for each question. The available pieces of information will be specified for each question on the computer screen. The type of information changes once after the fourth question.

## Calculation of your income in phase II

To compute your income in phase II, for each of the 8 questions one payoff-relevant round will be randomly selected by the computer. Exactly as in phase I, your profit is based on the distance of your estimation to the true value. The closer you are to the true value, the more money you earn (see definition of score in instructions of phase I). Please note that, even if you play in group mode, solely your own estimation affects your profit.

Your income from phase II is derived from the sum of points you have collected for each question in the corresponding payoff-relevant round, whereas the exchange rate is still 1 point corresponds to $\mathbf{0 , 3 0} €$. The maximal possible income in phase II is $38,40 €$.

## Total income

Your total income from the experiment consists of the guaranteed $5 €$, plus your income from Phase I, plus your income from phase II.

Your payoff as well as the correct answers will be provided by the end of the experiment.

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[^0]:    ${ }^{1}$ We start with B since there is already an appendix following the main text.

[^1]:    ${ }^{2}$ In this mathematical appendix, we use capital letters to indicate random variables.
    ${ }^{3} \sigma(\cdot)$ denotes the result of combining information, technically, it is the smallest sub- $\sigma$-algebra of $\mathcal{F}$ with respect to which all combined information is measurable.

[^2]:    ${ }^{4}$ In section 5.1, we study one baseline model, called the Sticking Model, in which this assumption is not satisfied. In that model, we have $g_{i i}=1$ for all $i=1, \ldots, 4$ and hence $g_{i j}=0$ for $i \neq j$.

[^3]:    ${ }^{5}$ Like the normal distribution, which is a standard functional form for beliefs on the unbounded real numbers, it is determined by two parameters only.
    ${ }^{6}$ The formal framework is provided in subsection B.4.1 below.

[^4]:    ${ }^{7}$ Overprecision, as it is called by Moore and Healy (2008), is also known as "judgmental overconfidence" (Herz et al., 2014), "overconfidence in interval estimates" (Soll and Klayman, 2004), or "resoluteness" (Bolton et al., 2013), and is defined as "excessive certainty regarding the accuracy of one's belief." Conservatism means that agents are not willing to learn sufficiently from new signals (e.g., Peterson and Beach, 1967; Möbius et al., 2011; Mannes and Moore, 2013; Ambuehl and Li, 2018). Of course, the two patterns are closely related to each other.
    ${ }^{8}$ Or, alternatively: agents learn from their neighbors, but they attach higher uncertainty to the beliefs of others than to their own belief.
    ${ }^{9}$ In the conservatism models (consisting of the specifications Standard-Plus and Sophisticated-Plus), we make assumptions about higher-order beliefs that close the model in the sense that no agent will expect another agent to behave in a different manner than in the one observed. In particular, we assume that all agents think of all other agents as overprecise; and that all agents think that all agents think that all agents are overprecise. In that way, an agent $i$ is not surprised that $j$ discounts $i$ 's behavior from $i$ 's point of view (from a neutral point of view, $j$ takes $i$ 's behavior as he should) and that $j$ overvalues $j$ 's guess (from $i$ 's and a neutral standpoint).

[^5]:    ${ }^{10}$ This number of observations may be some fixed integer, $n$, or, more generally, a random variable $N$ taking integer values.
    ${ }^{11}$ If the number of observations is a random variable, then one also conditions on $N=n$, and $P(N=n)$. appears as an additional factor.

[^6]:    ${ }^{12}$ A payoff function that is sufficiently convex elicits the mode of the belief if a subject's belief is uni-modal. This insight is not restricted to the beta distribution, but we do assume that agents maximize expected payoffs. Strong risk aversion and multi-modal distributions could alter this conclusion. In the experiment, the payoff function is discrete and it is not a priori clear whether it is "sufficiently convex". Numerical simulations with our specific payoff function and the beta distribution, however, validate that the mode is a very good approximation for the payoff-maximizing answer.

[^7]:    ${ }^{13}$ In our empirical application, $\tau$ is fixed to $\tau=5$, in order to appropriately account for the overprecision inherent in the confidence intervals given by the team members.

[^8]:    ${ }^{14}$ Grimm and Mengel (2018) propose another specification of the DeGroot weights. However, their extension does not lead to an additional prediction here because weights depend on the clustering coefficient, which is zero for all agents in the star network.

[^9]:    ${ }^{15}$ Interpretations for the cause of conservatism include forms of overprecision or kinds of anchoring bias in which the initial guess serves as anchor and the adjustments to the others' guesses is limited by parameter $\alpha$.

[^10]:    ${ }^{16}$ Two participants had to be excluded from payment for the use of a mobile phone. Whether they intended to cheat in the task or used their phones for other purposes is not known. The experimental results do not rely on decisions of these two subjects. Their decisions are kept in the results reported in the paper.

