Supplementary Online Material (SOM)

This supplementary online material belongs to the paper "The Swing Voter's Curse in Social Networks" by Berno Buechel and Lydia Mechtenberg. It consists of the following sections:[1]

B Supplementary Mathematical Appendix

B.1 A Necessary and Sufficient Condition

Proposition B.1. *In the specific model introduced in section 2, let m be odd and $\sum_j d_j =: l$ be even. The sincere strategy profile $\hat{\sigma}$ is an equilibrium if and only if the following conditions hold.*

1. *If $\exists i \in N$ with $d_i = 0$, then* $\sum_{x=1,3,...,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) [\nu(x,1) - \nu(-x,1)] \geq 0$, *where $\nu(x,1)$ denotes the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which are of size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.*[2]

2. *$\forall d_j \in \{d_1, ..., d_m\}$ such that $d_j > 0$ and for all $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$ the following holds:*

   (i) if $\bar{y}$ even, then $\sum_{x=-m+2,-m+4,...,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \sum_{y=1,3,...,\bar{y}-1} \nu(x, y|d_j) \geq 0$, *and*

   (ii) if $\bar{y}$ odd, then $\sum_{x=-m+2,-m+4,...,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left[\sum_{y=1,3,...,\bar{y}-2} \left(\nu(x, y|d_j) + \frac{1}{2}\nu(x, \bar{y}|d_j) \right) \right] \geq 0,$

   where $\nu(x, y|d_j)$ denotes the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which include element d_j+1, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

Proof. Part I shows necessity; part II shows sufficiency.

[1] We start with B since there is already an appendix A following the main text.

[2] In a multiset the same numbers can occur several times. In full analogy to the notion of a subset, we call a multiset that is contained in another multiset a "sub-multiset."

Part I. "ONLY IF". Suppose $\hat{\sigma}$ is an equilibrium. We show that the two conditions of Prop. B.1 are satisfied.

1. Since $\hat{\sigma}$ is an equilibrium, no player can beneficially deviate. In particular, if there is a non-expert $i \in N$ without a link, i.e., the qualification of the first condition of Prop. B.1 holds, then for any deviation $\sigma_i' \in \Sigma_i' = \{A, B\}$, we have $EU(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU(\sigma_{-i}, \sigma_i')$. W.l.o.g. suppose that $\sigma_i' = B$. Letting y denote the outcome under $\hat{\sigma}$ defined as the number of votes for A minus the number of votes for B, we observe that the deviation reduces the outcome y by one vote (because i votes for B instead of abstaining). The deviation σ_i' thus only affects the outcome if $y = +1$ and turns it into $y' = 0$ (i.e., if A wins by one vote under $\hat{\sigma}$, while there is a tie under $\sigma' := (\hat{\sigma}_{-i}, \sigma_i')$). Restricting attention to these draws of nature, we must still have that the sincere strategy profile leads to higher expected utility since it is an equilibrium by assumption:

$$EU_{|y=1}(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU_{|y=1}(\hat{\sigma}_{-i}, \sigma_i') = \frac{1}{2}. \tag{B.1}$$

   The right-hand side (RHS) is $\frac{1}{2}$ because this is the expected utility of a tie. Some more notation is helpful. Let x denote a distribution of signals defined as the number of A^*-signals minus the number of B^*-signals received by all experts. Let $P(x|A)$ denote the likelihood that the signals are x when the true state is A, and likewise for $P(x|B)$. Let $\hat{P}(x, y)$ designate the probability that signals x lead to outcome y under $\hat{\sigma}$. Then we can rewrite inequality B.1 as

$$\frac{\frac{1}{2}\sum_{x=-m,-m+2,\ldots,m} P(x|A)\hat{P}(x,1)}{\frac{1}{2}\sum_{x=-m,-m+2,\ldots,m}\Big(P(x|A)\hat{P}(x,1) + P(x|B)\hat{P}(x,1)\Big)} \geq \frac{1}{2}, \tag{B.2}$$

   since the expected utility under $\hat{\sigma}$ when restricting attention to the draws of nature that lead to a win of A by one vote equals the probability that A is true under these conditions.

   This simplifies to

$$\sum_{x=-m,-m+2,\ldots,m} P(x|A)\hat{P}(x,1) \geq \sum_{x=-m,-m+2,\ldots,m} P(x|B)\hat{P}(x,1) \tag{B.3}$$

   and further to

$$\sum_{x=-m,-m+2,\ldots,m} \left(P(x|A) - P(x|B)\right)\hat{P}(x,1) \geq 0. \tag{B.4}$$

   Now, we split the sum into positive and negative values of x and finally rejoin them by using $P(x|A) = P(-x|B)$:

$$\begin{aligned}
\sum_{x=-m,-m+2,\ldots,m} (P(x|A) - P(x|B))\,\hat{P}(x,1) &\geq 0 \\
\Leftrightarrow \sum_{x=1,3,\ldots,m} (P(x|A) - P(x|B))\,\hat{P}(x,1) & \\
+ \sum_{x=-m,-m+2,\ldots,-1} (P(x|A) - P(x|B))\,\hat{P}(x,1) &\geq 0 \\
\Leftrightarrow \sum_{x=1,3,\ldots,m} (P(x|A) - P(x|B))\,\hat{P}(x,1) & \\
+ \sum_{x=1,3,\ldots,m} (P(-x|A) - P(-x|B))\,\hat{P}(-x,1) &\geq 0 \\
\Leftrightarrow \sum_{x=1,3,\ldots,m} (P(x|A) - P(x|B))\,\hat{P}(x,1) & \\
+ \sum_{x=1,3,\ldots,m} (P(x|B) - P(x|A))\,\hat{P}(-x,1) &\geq 0 \\
\Leftrightarrow \sum_{x=1,3,\ldots,m} (P(x|A) - P(x|B))\,[\hat{P}(x,1) - \hat{P}(-x,1)] &\geq 0 \\
\Leftrightarrow \sum_{x=1,3,\ldots,m} (P(x|A) - P(-x|A))\,[\hat{P}(x,1) - \hat{P}(-x,1)] &\geq 0.
\end{aligned}$$

Independent of the strategy profile, $P(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}}$. For a draw of signals with difference x (in numbers of A^*-signals and B^*-signals), the outcome $y = +1$ is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+1}{2}$ votes for A. All of the A-votes under $\hat{\sigma}$ can be partitioned such that each element of the partition is referred to an expert j with signal A^*. Such an expert accounts for $d_j + 1$ votes because there is her vote and the votes of her audience. Hence, the probability that draw of nature x leads to outcome $y = +1$ is determined by the frequency with which $\frac{m+x}{2}$ experts who have received signal A^* account for exactly $\frac{m+l+1}{2}$ votes. This frequency is given by the number of "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which have size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.

Considering all possible allocations of $\frac{m+x}{2}$ A^*-signals among m experts, there are $\binom{m}{\frac{m+x}{2}}$ possibilities (which is the number of all "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome $y = +1$ is

$$\hat{P}(x,+1) = \frac{\nu(x,1)}{\binom{m}{\frac{m+x}{2}}},$$

where $\nu(x,1)$ denotes the number of "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ of size $\frac{m+x}{2}$ and sum $\frac{m+l+1}{2}$.

Plugging the equations for $P(x|A)$ and $\hat{P}(x,1)$ into the inequality derived above yields:

$$\sum_{x=1,3,\dots,m}\left(\binom{m}{\frac{m+x}{2}}p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}-\binom{m}{\frac{m-x}{2}}(1-p)^{\frac{m+x}{2}}p^{\frac{m-x}{2}}\right) \cdot\left[\frac{\nu(x,1)}{\binom{m}{\frac{m+x}{2}}}-\frac{\nu(-x,1)}{\binom{m}{\frac{m-x}{2}}}\right]\geq 0. \tag{B.5}$$

Since $\binom{m}{\frac{m-x}{2}}=\binom{m}{\frac{m+x}{2}}$, these factors cancel out such that we get

$$\sum_{x=1,3,\dots,m}\left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}-(1-p)^{\frac{m+x}{2}}p^{\frac{m-x}{2}}\right)[\nu(x,1)-\nu(-x,1)]\geq 0. \tag{B.6}$$

This shows that the first condition of Prop. B.1 is indeed implied by the assumption that $\hat{\sigma}$ is an equilibrium.

2. Let us turn to the second condition of Prop. B.1 by considering some expert $j \in M$ with $d_j > 0$. W.l.o.g. let her signal be A^*. Under the sincere strategy profile j will vote and communicate her signal, i.e., A. Abstention reduces the outcome y by one vote, voting the opposite reduces the outcome y by two votes. Sending no message reduces the outcome by d_j votes. Sending the opposite message reduces the outcome by $2d_j$ votes. Therefore, there are feasible deviations for j that reduce the outcome by a number of votes $\bar{y}$ which is in the following set $\{1,2,d_j,d_j+1,d_j+2,2d_j,2d_j+1,2d_j+2\}$.

   By the assumption that $\hat{\sigma}$ is an equilibrium, there is no beneficial deviation for j. That is, for any deviation $\sigma'_j \in \Sigma'_j$, we have $EU^{s_j=A^*}(\hat{\sigma}_{-j},\hat{\sigma}_j) \geq EU^{s_j=A^*}(\hat{\sigma}_{-j},\sigma'_j)$. Considering some deviation σ'_j and the corresponding reduction of the outcome by $\bar{y}$, the implemented alternatives only differ for draws of nature such that $y>0$ and $y' \leq 0$, i.e for outcomes y such that $0<y\leq\bar{y}$ (because only then the reduction of support for the received signal has any effect). Therefore, the inequality of expected utility must also hold when focusing on these cases, i.e.

$$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j},\hat{\sigma}_j)\geq EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j},\sigma'_j). \tag{B.7}$$

   (i) Suppose first that $\bar{y}$ is even. Then the deviation σ'_j turns all outcomes in which A wins and $0<y\leq\bar{y}-1$ into a win of alternative B (outcomes $y=\bar{y}$ are not possible because y is odd). Therefore, the expected utility of strategy profile $\hat{\sigma}$ (respectively, $\sigma' := (\hat{\sigma}_{-j},\sigma'_j)$), focusing on these cases, is the probability that A (respectively, B) is true in these cases. Let $P_{s_j=A^*}(x|\omega=A) =: P_A(x|A)$ denote the probability that the signal distribution is x and that expert j has received signal A^* when the true state is A, and similarly for $P_{s_j=A^*}(x|\omega=B) =: P_A(x|B)$. Moreover, let $\hat{P}_{s_j=A^*}(x,y) =: \hat{P}_A(x,y)$ be the probability that the signals x lead to outcome y under $\hat{\sigma}$, given that expert j has received signal A^*. Note that $\hat{P}_A(x,y)$ is not defined for $x=-m$ because if all experts have received signal B^* it is not possible that expert j has received signal A^*.

Then we can rewrite inequality B.7 as

$$\begin{aligned}
&\sum_{x=-m+2,-m+4,\ldots,m} P_A(x|A) \sum_{y=1,3,\ldots,\bar{y}-1} \hat{P}_A(x,y) \geq \\
&\sum_{x=-m+2,-m+4,\ldots,m} P_A(x|B) \sum_{y=1,3,\ldots,\bar{y}-1} \hat{P}_A(x,y).
\end{aligned} \tag{B.8}$$

inequality B.8 incorporates that the likelihood of A being true is greater or equal than the likelihood of B being true given that the deviation is effective and that expert j has received signal A^*.[3] This inequality simplifies to

$$\begin{aligned}
&\sum_{x=-m+2,-m+4,\ldots,m} (P_A(x|A) - P_A(x|B)) \\
&\cdot \sum_{y=1,3,\ldots,\bar{y}-1} \hat{P}_A(x,y) \geq 0.
\end{aligned} \tag{B.9}$$

Independent of the strategy profile, $P_A(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} \cdot \frac{\frac{m+x}{2}}{m}$ and $P_A(x|B) = \binom{m}{\frac{m-x}{2}} p^{\frac{m-x}{2}}(1-p)^{\frac{m+x}{2}} \cdot \frac{\frac{m+x}{2}}{m}$. The factor before the multiplication sign is the probability that there are exactly $\frac{m+x}{2}$ A^*-signals. Given such a distribution, the factor after the multiplication sign is the probability that expert j has received signal A^*.

For a distribution of signals x, the outcome y is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+y}{2}$ votes for A. All of the A-votes under $\hat{\sigma}$ can be partitioned such that each element is referred to an expert k with signal A^*. Such an expert accounts for $d_k + 1$ votes (because there is her vote and the votes of her audience). By assumption, expert j has received signal A^* and thus there are at least d_j+1 votes for A under $\hat{\sigma}$. The probability that draw of nature x leads to outcome y is determined by the frequency that the $\frac{m+x}{2}$ experts who have received signal A^* account for exactly $\frac{m+l+y}{2}$ votes. Hence, this frequency is given by the number of "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$. Considering all possible allocations of $\frac{m+x}{2}$ A^*-signals among m experts such that j also receives signal A^*, there are $\binom{m-1}{\frac{m+x}{2}-1}$ possibilities (which is the number of all "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which include element $d_j + 1$ and are of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome y, given that expert j has received signal A^*, is

$$\hat{P}_A(x,y) = \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}},$$

where $\nu(x,y|d_j)$ denotes the number of "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

[3] To get the absolute probabilities of A (respectively B) being true, we can divide the LHS (respectively the RHS) of inequality B.8 by the sum of the LHS and the RHS.

Hence, we can rewrite inequality B.9 as follows

$$
\begin{aligned}
\sum_{x=-m+2,-m+4,\ldots,m} (P_A(x|A) - P_A(x|B)) \sum_{y=1,3,\ldots,\bar{y}-1} \hat{P}_A(x,y) &\geq 0 \\
\Leftrightarrow \sum_{x=-m+2,-m+4,\ldots,m} \left(\binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \right. & \\
\left. - \binom{m}{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \right) \sum_{y=1,3,\ldots,\bar{y}-1} \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} &\geq 0 \\
\Leftrightarrow \sum_{x=-m+2,-m+4,\ldots,m} \binom{m}{\frac{m+x}{2}} \frac{\frac{m+x}{2}}{m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) & \\
\cdot \sum_{y=1,3,\ldots,\bar{y}-1} \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} &\geq 0.
\end{aligned}
$$

We have used that $\binom{m}{\frac{m+x}{2}} = \binom{m}{\frac{m-x}{2}}$. Finally, we observe that the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{\frac{m+x}{2}}{m}$, and $\frac{1}{\binom{m-1}{\frac{m+x}{2}-1}}$ simplify to one because $\frac{\binom{m}{\frac{m+x}{2}}}{\binom{m-1}{\frac{m+x}{2}-1}} = \frac{m}{\frac{m+x}{2}}$ such that we get

$$
\sum_{x=-m+2,-m+4,\ldots,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \sum_{y=1,3,\ldots,\bar{y}-1} \nu(x,y|d_j) \geq 0 \tag{B.10}
$$

We have shown that inequality B.10, which coincides with condition 2(i) of Prop. B.1, holds for any $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$ even.

(ii) Suppose now that $\bar{y}$ is odd. (Still, we keep the assumption that some expert $j \in M$ with $d_j > 0$ has received signal A^* and considers a deviation σ'_j that reduces the outcome by $\bar{y}$). Then the deviation σ'_j turns all outcomes in which A wins and $0 < y \leq \bar{y}$ into a win of alternative B for $y = 1, 3, ..., \bar{y} - 2$ and into a tie for $y = \bar{y}$. Therefore,

$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j}, \sigma'_j) = \frac{\sum_{x=-m+2,-m+4,\ldots,m} \left(P_A(x|B)\left(\sum_{y=1,3,\ldots,\bar{y}-2} P_A(x,y) + \frac{1}{2}\hat{P}_A(x,\bar{y})\right) + \frac{1}{2}P_A(x|A)\hat{P}_A(x,\bar{y})\right)}{\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)+P_A(x|B))\sum_{y=1,3,\ldots,\bar{y}} \hat{P}_A(x,y)}$.

The denominator is the probability that an outcome under $\hat{\sigma}$ is reached such that the deviation has some effect. The numerator consists of the probability that B is true for the cases where the deviation leads to a win of alternative B and of half the probabilities that A or B are true when the deviation leads to a tie.

The expected utility of the sincere strategy profile amounts to

$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j}, \hat{\sigma}_j) = \frac{\sum_{x=-m+2,-m+4,\ldots,m} P_A(x|A)\left(\sum_{y=1,3,\ldots,\bar{y}-2} \hat{P}_A(x,y) + \hat{P}_A(x,\bar{y})\right)}{\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)+P_A(x|B))\sum_{y=1,3,\ldots,\bar{y}} \hat{P}_A(x,y)}$.

The numerator is the probability that A is true under the cases where the deviation has some effect. Since the denominator is the same as above, we can rewrite inequality B.7 as

$\sum_{x=-m+2,-m+4,\ldots,m} \left(P_A(x|A)\left(\sum_{y=1,3,\ldots,\bar{y}-2} \hat{P}_A(x,y) + \hat{P}_A(x,\bar{y})\right) - P_A(x|B)\right.$
$\left.\cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2} \hat{P}_A(x,y) + \frac{1}{2}\hat{P}_A(x,\bar{y})\right) - \frac{1}{2}P_A(x|A)\hat{P}_A(x,\bar{y})\right) \geq 0$ and further simplify

it to

$$\sum_{x=-m+2,-m+4,\ldots,m} \left(P_A(x|A) - P_A(x|B)\right) \cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2} \hat{P}(x,y|d_j) + \frac{1}{2}\hat{P}_A(x,\bar{y}) \right) \geq 0. \tag{B.11}$$

Now, we plug in $P_A(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m}$ and $P_A(x|B) = \binom{m}{\frac{m-x}{2}} p^{\frac{m-x}{2}}(1-p)^{\frac{m+x}{2}} \frac{\frac{m+x}{2}}{m}$; as well as $\hat{P}_A(x,y) = \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}}$. This yields:

$$\sum_{x=-m+2,-m+4,\ldots,m} \binom{m}{\frac{m+x}{2}} \frac{\frac{m+x}{2}}{m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}}\right) \cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2} \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} + \frac{1}{2}\frac{\nu(x,\bar{y}|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} \right) \geq 0. \tag{B.12}$$

Again, the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{\frac{m+x}{2}}{m}$, and $\frac{1}{\binom{m-1}{\frac{m+x}{2}-1}}$ cancel out since their product is 1. Hence, inequality B.12 becomes

$$\sum_{x=-m+2,-m+4,\ldots,m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}}\right) \cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2} \nu(x,y|d_j) + \frac{1}{2}\nu(x,\bar{y}|d_j) \right) \geq 0. \tag{B.13}$$

Inequality B.13 holds for any $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$ odd and coincides with condition 2(ii) of Prop. B.1.

We have derived the implications for an arbitrary expert with degree $d_j > 0$ and for some arbitrary $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$. The derived conditions 2(i) and 2(ii) must hence hold for any $d_j \in \{d_1, \ldots, d_m\}$ such that $d_j > 0$. For the case of the empty network, in which no single expert has an audience, the strategy profile $\hat{\sigma}$ is not interesting to study because communication is impossible, but formally still Prop. B.1 applies. In this special case condition 2 is trivially satisfied. Thus, we have shown that if $\hat{\sigma}$ is an equilibrium, then the second condition of Prop. B.1 is also satisfied.

Part II. "IF". Suppose that the two conditions of Prop. B.1 are satisfied. We show that $\hat{\sigma}$ is an equilibrium by deriving the implications of these two conditions for every kind of player.

- Non-experts without a link: Consider any non-expert $i \in N$ with $d_i = 0$. The set of strategies is $\{A, B, \phi\}$ and $\hat{\sigma}_i = \phi$. Suppose condition 1 of Prop. B.1 holds, which is inequality B.6. In part I of the proof we used a sequence of transformations to rewrite inequality B.1 as inequality B.6. Since these were all equivalence transformations, the assumption that inequality B.6 holds implies that inequality B.1 holds. Thus, condition 1 of Prop. B.1 implies that for a non-expert without a link deviating from $\hat{\sigma}$ does not increase expected utility, given that the outcome is $y = +1$, i.e., given that the deviation has any effect on the outcome.

- Experts with an audience: Consider any expert $j \in M$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals. To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A^* because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to nine. Consider any deviation σ'_j. This deviation reduces the voting outcome y that is attained under $\hat{\sigma}$ by a number $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$. For each of these numbers conditions 2(i) and 2(ii) of Prop. B.1 are equivalent to inequality B.7 since the conditions 2(i) and 2(ii) were derived by equivalence transformations of inequality B.7. Thus, for any deviation of an expert with an audience, the expected utility is weakly smaller than under $\hat{\sigma}$, when restricting attention to the cases where the deviation has some effect on the outcome and hence in general as well.

- Experts without an audience: Consider any expert $j \in M$ with $d_j = 0$. W.l.o.g. assume that j has received signal A^*. Under $\hat{\sigma}$ expert i would vote A. Alternatively, she can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (These deviations have already been considered for experts with an audience when letting $\bar{y} = 2$, respectively, $\bar{y} = 1$.) These deviations are not increasing expected utility since condition 2(i) of Prop. B.1 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. B.1 holds in particular for $\bar{y} = 1$ such that inequality B.7 is satisfied.

- Non-experts with a link: Consider any non-expert $i \in N$ with $d_i = 1$. W.l.o.g. assume that i has received message A. Under $\hat{\sigma}$ non-expert i votes A. Alternatively, he can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (The effect of these two deviations is as if an expert with signal A^* would vote for B respectively abstain.) Again, since condition 2(i) of Prop. B.1 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. B.1 holds in particular for $\bar{y} = 1$, inequality B.7 is satisfied such that these deviations do not increase expected utility.

We have shown in part II of the proof that the conditions 1 and 2 provided in Prop. B.1 imply that no player can beneficially deviate from $\hat{\sigma}$. □

B.2 Equilibrium Analysis of Examples 1, 2, and 3

We define the concept of a *transmission network* $g^* \subseteq g$ as follows: A link g^*_{ij} between non-expert $i \in N$ and expert $j \in M$ exists if and only if j truthfully transmits her signal to i.

Truthful transmission requires that (1) the expert sends a message $m_j^* \in \{A, B, \emptyset\}$ whenever her signal is A^* and sends a different message $m_j^{*\prime} \in \{A, B, \emptyset\}, m_j^{*\prime} \neq m_j^*$ whenever her signal is B^*; and that (2) the posterior belief of the non-expert, conditional on the message received, equals the posterior belief of the expert, conditional on her signal. In equilibrium, (1) implies (2). A transmission network g^* arises in the communication stage on the equilibrium path. Note that different communication strategies support a given g^*, e.g., sending message A after signal A^* and message B after signal B^* transmits the same information as sending message B after signal A^* and message A after signal B^*. Since we are only interested in the information transmission (and voting behavior) in equilibrium and not in the precise "language" that transmits the information, we will not fully specify the communication strategies but refer to the resulting transmission network instead. Hence, we can drop any explicit reference to the full strategy profiles σ. Let v denote the strategy profile of all players on the voting stage. Then, any type of equilibrium of our Examples 1,2, and 3 can be fully characterized by g^* and v. Note that any two equilibria that are characterized by a given g^* and v are identical with respect to all equilibrium beliefs, voting strategies and outcomes.[4]

Let $\widetilde{m_i}(s_j) \in \{A, B, \emptyset\}$ denote the meaning that non-expert i ascribes to message m_j^* if $g_{ij}^* = 1$ for some expert j who received signal $s_j \in \{A^*, B^*\}$: i believes that the expert's vote recommendation is $\widetilde{m_i}$, with $\widetilde{m_i} = A$ indicating a recommendation to vote for A, $\widetilde{m_i} = B$ indicating a recommendation to vote for B, and $\widetilde{m_i} = \emptyset$ indicating a recommendation to abstain. Slightly abusing notation, we write $v_i(\widetilde{m_i}) \in \{A, B, \emptyset\}$ to denote the voting strategy of non-expert i with $g_{ij}^* = 1$ for some j. Analogously, the voting strategy of a non-expert i with $g_{ij}^* = 0$ for all $j \in M$ is denoted by $v_i(\emptyset) \in \{A, B, \emptyset\}$. Note that $\tilde{m}_i = \emptyset$ implies $g_{ij}^* = 0$ and $g_{ij} = 1$ in the three examples. Let $\widetilde{s_l}$ denote either signal $s_l \in \{A^*, B^*\}$ received by $l \in M$ or the meaning $\widetilde{m_l}$ of the message received by $l \in N$. Then, we write $v_l(\widetilde{s_l}) \in \{A, B, \emptyset\}$ to denote the voting strategy of $l \in M \cup N$.

We now define the following four selection criteria that guide our equilibrium analysis:

1. **Purity**: The equilibrium is in pure strategies.
2. **Symmetry**: Any two experts, as well as any two non-experts, with the same degree in the transmission network apply identical strategies.
3. **Monotonicity**: If $v_i\left(\widetilde{m_i}'\right) = \widetilde{m_i}$ for some $\widetilde{m_i}' \in \{A, B, \emptyset\}$, then $v_i(\widetilde{m_i}) = \widetilde{m_i}$; and if $\widetilde{m_i}\left(s_j'\right) = s_j$ for some $s_j' \in \{A, B\}$, then $\widetilde{m_i}(s_j) = s_j$.
4. **Neutrality**: (i) Unbiased voting: Either $v_l(\widetilde{s_l}) = \widetilde{s_l}$ for all $\widetilde{s_l} \in \{A, B\}$ or $v_l(\widetilde{s_l}) \neq \widetilde{s_l}$ for all $\widetilde{s_l} \in \{A, B\}$; and $v_i(\emptyset) = \emptyset$. (ii) Unbiased information transmission: Either $\widetilde{m_i}(s_j) = s_j$ for all $s_j \in \{A, B\}$, or $\widetilde{m_i}(s_j) = \emptyset$ (i.e. $g_{ij}^* = 0$) for all $s_j \in \{A, B\}$.

We now define a voting strategy profile v for any transmission network g^* as follows: Order the experts according to their degrees d_j^* in g^* in decreasing order, indicate the experts with the highest degree in the transmission network by the index δ_1^* and the experts with the

[4] We do not explicitly specify off-equilibrium beliefs; hence the equilibria of one type may differ in those. However, equating the off-equilibrium belief with the priors for any non-expert who, surprisingly, finds himself uninformed after an expert's deviation from g^* on the communication stage supports all selected equilibria.

second-highest degree with the index δ_2^*, etc. Indicate the lowest degree of experts by index δ_M^* and the lowest possible degree of non-experts by index $\delta_N^* = 0$.[5] Order the non-experts according to their degrees d_i^* in decreasing order, indicate the non-experts with degree one in the transmission network by the index 1 and the non-experts with degree zero with the index 0. Then, a strategy profile on the voting stage is given by

$$v = \left\{ \begin{array}{c} v_{\delta_1}(A), v_{\delta_1}(B); v_{\delta_2}(A), v_{\delta_2}(B); ..., v_{\delta_M}(A), v_{\delta_M}(B); \\ v_1(A), v_1(B); v_0(A), v_0(B), v_0(\emptyset) \end{array} \right\}.$$

Note that a deviation of some expert j from g^* on the communication stage is either a lie that cannot be identified as such (i.e. $v_0(A) = v_1(A)$ and $v_0(B) = v_1(B)$) or an empty message. Hence, in what follows we can drop $v_0(A)$ and $v_0(B)$ as elements of the strategy profiles.

B.2.1 Example 1

In Example 1, we have two possibilities. Either the transmission network is empty due to a babbling equilibrium. Then, the strategy profiles conforming to our selection criteria imply that either all experts abstain or all experts vote their signal while all non-experts abstain. The latter strategy profile is a "let the experts decide (LTED)" equilibrium. This is an equilibrium in every game and we do not discuss it further in this analysis. The second possibility is that $r \in \{1, 2, 3, 4\}$ experts transmit their signal to the non-expert linked to them, while the remaining experts do not. (Note that we fully characterize g^* by r in this example.) Hence, there are two possible types of experts and two types of non-experts: those with degree $d_l^* = 1$ and those with $d_l^* = 0$. Hence, the strategy profiles on the voting stage are of the form

$$v = \{v_1(A), v_1(B); v_2(A), v_2(B); v_1(A), v_1(B); v_0(\emptyset)\}.$$

The strategy profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result that we state without proof.[6]

[5] The lowest degree of non-experts is zero off equilibrium, even though it might be one on the equilibrium path.

[6] The proof of this and all other propositions in this subsection can be obtained by the authors upon request.

Proposition B.2. *Strategy profile v_1 and $r \in \{3,4\}$ are (sincere) equilibria; v_2 and $r \in \{1,2,3,4\}$ are (LTED) equilibria; v_3 and $r \in \{1,3\}$ are equilibria (with sincere voting and expert abstention); v_4 and $r \in \{1,3\}$ are ("let some experts decide") equilibria; v_5 and $r \in \{1,2,3,4\}$ are (delegation) equilibria and outcome-equivalent to σ^*; v_6 and $r \in \{2,4\}$ are ("let some experts decide") equilibria; v_7 and $r \in \{1,3\}$ are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure B.1.

B.2.2 Example 2

Again, we have two possibilities. Either the transmission network is empty due to a babbling equilibrium and a LTED equilibrium exists. The second possibility is that the center of the star (expert 1) transmits her signal to all non-experts. We now consider this second possibility and refer to the resulting transmission network as g_2^*. The strategy profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

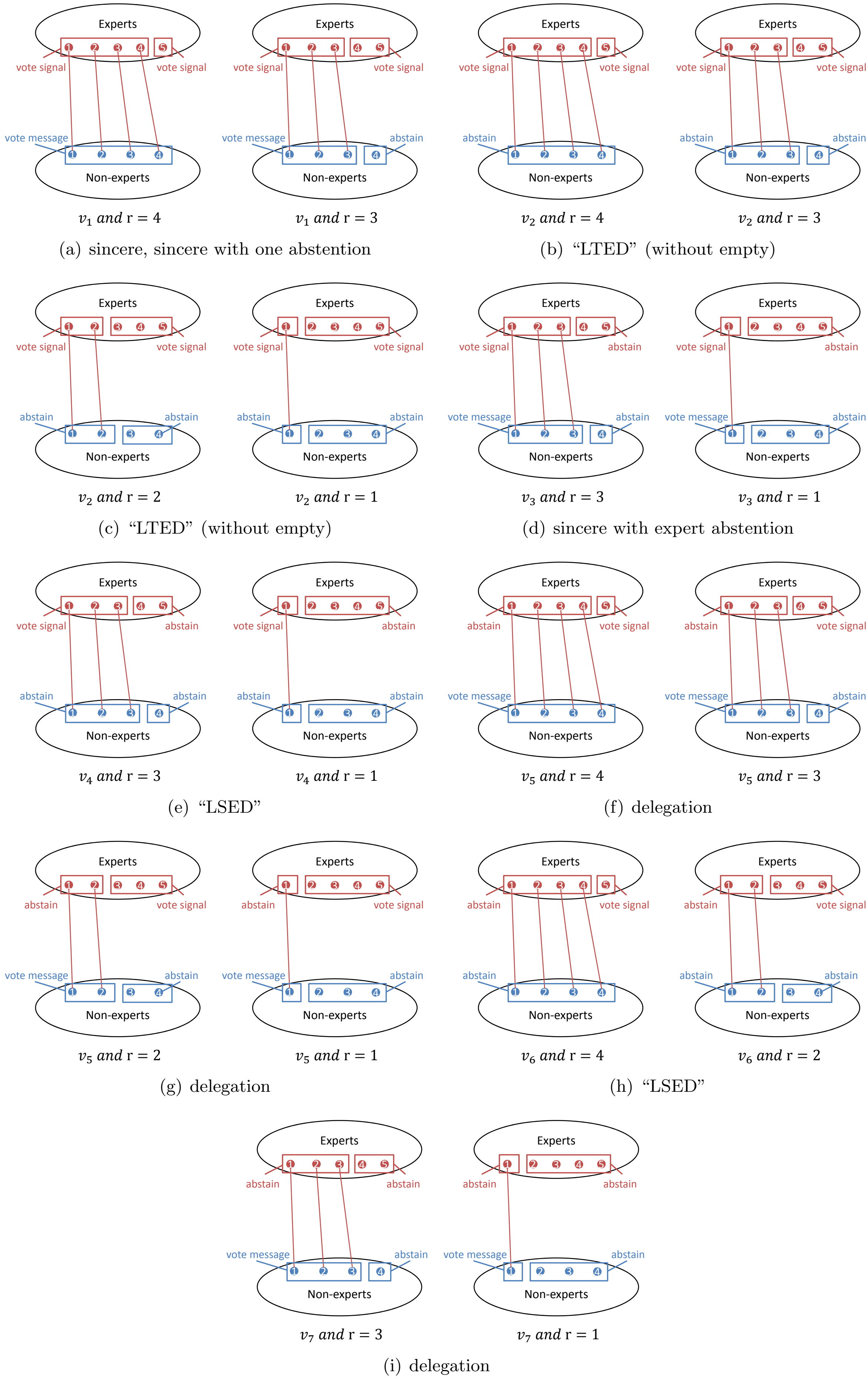
Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result.

Proposition B.3. *Strategy profile v_2 and g_2^* are (LTED) equilibria; v_3 and g_2^* are equilibria (with sincere voting and expert abstention); v_4 and g_2^* are ("let some experts decide") equilibria; v_7 and g_2^* are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure B.2.

B.2.3 Example 3

In this example we have three possibilities which reduce to two if we ignore the empty transmission network whose only equilibrium LTED has been discussed above. These two possibilities are the following: (1) Either $g_{ij} = g_{ij}^*$ for all $i, j \in N \cup M$; then, the two experts with degree two in g are symmetric, the four non-experts are symmetric, and the three experts with degree zero in g are symmetric. (2) Or degree $d_j = d_j^* = 2$ for exactly one expert j and $d_{j'}^* = 0$ for the other expert j' who has degree $d_{j'} = 1$ in g. Then, this other expert j' is symmetric to the experts with degree zero in g; the two non-experts i with $g_{ij}^* = 1$ are symmetric, and the two non-experts with $g_{ij}^* = 0$ are symmetric.



(a) sincere, sincere with one abstention

(b) "LTED" (without empty)

(c) "LTED" (without empty)

(d) sincere with expert abstention

(e) "LSED"

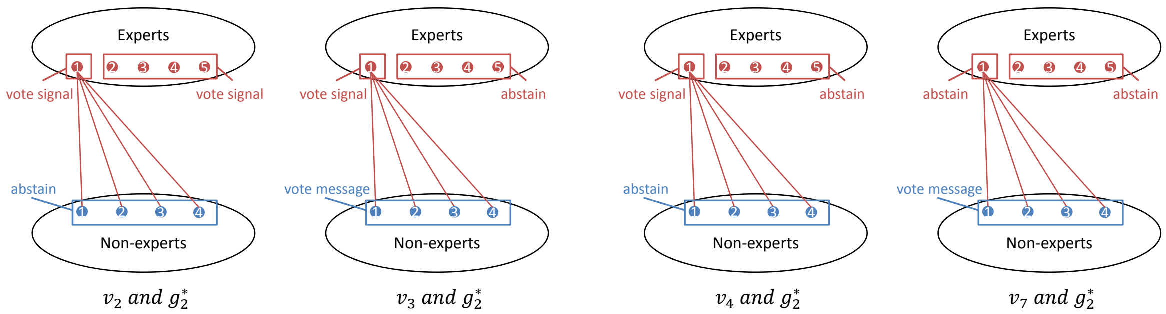
(f) delegation

(g) delegation

(h) "LSED"

(i) delegation

Figure B.1: All equilibria of Proposition B.2.



(a) "LTED" and sincere with expert abstention (b) "LSED" and delegation

Figure B.2: All equilibria of Proposition B.3.

Possibility (1). Let us first consider the case in which the transmission network equals the exogenous network; and let g_{31}^* denote this network. Then, the profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$$v_1 = \{A, B; A, B; A, B; \emptyset\},$$
$$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\},$$
$$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\},$$
$$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\},$$
$$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\},$$
$$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\},$$
$$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}, \text{ and}$$
$$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}.$$

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result.

Proposition B.4. *Strategy profile v_1 and g_{31}^* are (sincere) equilibria; v_2 and g_{31}^* are (LTED) equilibria; v_5 and g_{31}^* are (delegation) equilibria; v_6 and g_{31}^* are ("let some experts decide") equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure B.3 below.

Possibility (2). Let us now consider the case in which the transmission network differs from the exogenous network in that only one expert transmits his signal, and let us refer to this transmission network as g_{32}^*. Then, the profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

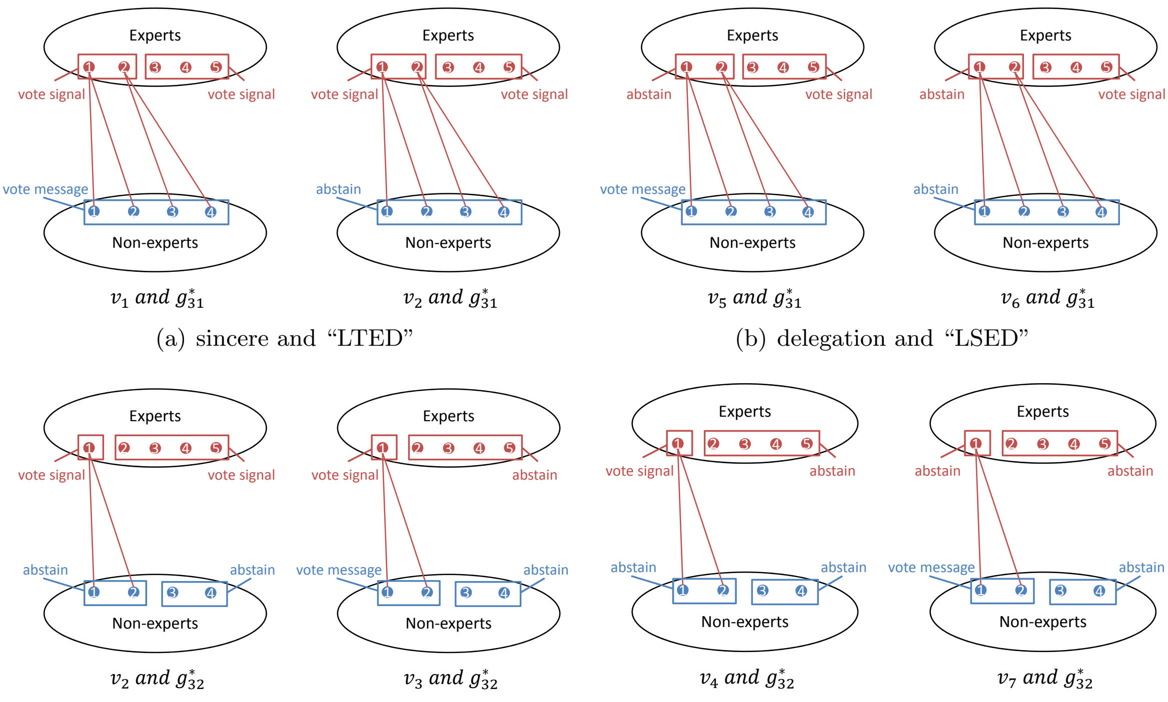
$$v_1 = \{A, B; A, B; A, B; \emptyset\},$$
$$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\},$$
$$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\},$$
$$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\},$$

$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result that we state without proof.

Proposition B.5. *Strategy profile v_2 and g^*_{32} are (LTED) equilibria; v_3 and g^*_{32} are equilibria (sincere voting with some experts abstaining); v_4 and g^*_{32} are ("let some experts decide") equilibria; v_7 and g^*_{32} are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure B.3.



(a) sincere and "LTED"
(b) delegation and "LSED"
(c) "LTED" and sincere with expert abstention
(d) "LSED" and delegation

Figure B.3: All equilibria of Propositions B.4 and B.5 .

B.3 Equivalence of Definitions 2.2 and A.1

Definitions 2.2 and A.1 both define the notions of strong and weak balancedness. We show here that Definition A.1 of the general model introduced in section A.2 applied to the specific model introduced in section 2 is indeed equivalent to Definition 2.2 and moreover that strong balancedness implies weak balancedness.

Formally, we consider the general model introduced in section A.2 and make the assumption that the set of voters V can be partitioned into a set of experts M who receive an informative signal of the homogenous quality $p_j = p > 0.5$ and a set of non-experts N who receive a non-informative signal of precision $p_i = 0.5$. The network structure g is bipartite such that there are only links between experts and non-experts. Moreover, audiences are non-overlapping, i.e., each non-expert is linked to at most one expert.

Notice that the neighborhood of an expert V_j consists of her audience of linked non-experts (if any). The neighborhood of a non-expert V_i consists of the linked expert (if any). Therefore, an expert $j \in M$ is a believer of a set $S \subseteq V$, i.e. $j \in V^+(S)$, if and only if $j \in S$; and a non-expert $i \in N$ is a believer of a set $S \subseteq V$, i.e. $i \in V^+(S)$, if and only if $j \in S$ for the linked expert j (with ij in g). Thus, for any set $S \subseteq V$, the set of believers $V^+(S)$ consists of the experts who are in S and of their audiences of non-experts. Hence

$$|V^+(S)| = |M \cap S| + \sum_{j \in (M \cap S)} d_j. \tag{B.14}$$

Notice also that the expertise of a set of voters $S \subseteq V$ is proportional to the number of experts in the set since $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) = |S \cap M| * \log(\frac{p}{1-p})$. Thus, a set of voters $S \subseteq V$ is better informed than the complementary set $V \setminus S$ if and only if S contains a majority of experts. Formally,

$$\sum_{j \in S} \log(\frac{p_j}{1-p_j}) > \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k}) \text{ if and only if } |M \cap S| > |M \setminus S|, \tag{B.15}$$

or equivalently

$$\prod_{j \in S} \frac{p_j}{1-p_j} > \prod_{k \in V \setminus S} \frac{p_k}{1-p_k} \text{ if and only if } |M \cap S| > \frac{m}{2}. \tag{B.16}$$

Strong balancedness. Strong balancedness according to Definition A.1 (a) is satisfied if and only if $\forall S \subseteq V$,

$$\prod_{j \in S} \frac{p_j}{1-p_j} > \prod_{k \in V \setminus S} \frac{p_k}{1-p_k} \text{ implies } |V^+(S)| > |V^-(S)|.$$

Since in the specific model $p_j = p$ for all $j \in M$ and $p_i = 0.5$ for all $i \in N$, and since $|V^+(S)| = |M \cap S| + \sum_{j \in (M \cap S)} d_j$, this is equivalent to $\forall S \subseteq V$,

$$|S \cap M| > \frac{m}{2} \text{ implies } |M \cap S| + \sum_{j \in (M \cap S)} d_j > |M \setminus S| + \sum_{k \in (M \setminus S)} d_k.$$

Since in a set S the non-experts $S \cap N$ do not matter for the above equations, the statement above is equivalent to $\forall M'' \subseteq M$,

$$m'' > \frac{m}{2} \text{ implies } m'' + \sum_{j \in M''} d_j > m - m'' + \sum_{k \in (M \setminus M'')} d_k. \tag{B.17}$$

If equation B.17 holds for a given set M'', then it also holds for a superset of it. Hence, for m odd, the condition above (which makes a requirement on all sets $M'' \subseteq M$ with $m'' > \frac{m}{2}$)

is equivalent to $\forall M'' \subseteq M$ such that $m'' = \frac{m+1}{2}$,

$$\frac{m+1}{2} + \sum_{j \in M''} d_j > \frac{m-1}{2} + \sum_{k \in (M \setminus M'')} d_k,$$

which simplifies to

$$1 + \sum_{j \in M''} d_j > \sum_{k \in (M \setminus M'')} d_k$$

and finally to

$$\sum_{j \in M''} d_j \geq \sum_{k \in (M \setminus M'')} d_k,$$

which is the definition of strong balancedness according to Definition 2.2.

Weak balancedness. Definition A.1 part (b) uses the following two notions. For a voter $i \in V$, $\mathcal{S}_i$ collects all sets of voters S, of which i is a believer, i.e. $i \in V^+(S)$, and which have slightly more believers than non-believers, i.e. $|V^+(S)| - |V^-(S)| \in \{0, 1, 2\}$. $\mathcal{Q}_i$ collects all subsets of these sets that belong to i's neighborhood, i.e. $\mathcal{Q}_i := \{Q \subseteq V | Q = (V_i \cup i) \cap S \text{ for some } S \in \mathcal{S}_i\}$.

Under the specific assumptions (that nest the model of section 2 in the framework of section A.2), these notions simplify as follows. For an expert $j \in M$, $\mathcal{S}_j$ collects all sets of voters S, that include expert j, i.e. $j \in S$, and whose experts together with their audiences have slightly more voters than the complementary set, i.e.

$$|M \cap S| + \sum_{k \in (M \cap S)} d_k - (|M \setminus S| + \sum_{l \in (M \setminus S)} d_l) \in \{0, 1, 2\},$$

which is equivalent to

$$\sum_{k \in (M \cap S)} (d_k + 1) - \sum_{l \in (M \setminus S)} (d_l + 1) \in \{0, 1, 2\}. \tag{B.18}$$

Moreover, for an expert $j \in M$, $\mathcal{Q}_j$ collects all subsets Q of these sets S that belong to i's neighborhood, which consists of the expert j herself and a (possibly emtpy) subset of her audience of linked non-experts, i.e. $j \in Q \subseteq \{V_i \cup j\}$. Hence, either $\mathcal{S}_j = \emptyset$, then $\mathcal{Q}_j = \emptyset$; or $\mathcal{S}_j \neq \emptyset$, then $\{\{j\}\} \in \mathcal{Q}_j$.

For a non-expert $i \in N$, $\mathcal{S}_i = \emptyset$ if $d_i = 0$ because $i \notin V^+(S)$ for any set S. If non-expert i is linked to some expert j, then $\mathcal{S}_i = \mathcal{S}_j$, i.e., the set $\mathcal{S}_i$ coincides with the corresponding set of the expert linked to non-expert i. Moreover, for a non-expert $i \in N$, $\mathcal{Q}_i$ collects all subsets of these sets that belong to i's neighborhood, which consists only of the expert j who is linked to i, i.e. $Q = \{\{j\}\}$. Hence, either $\mathcal{S}_i = \emptyset$ (e.g. because $d_i = 0$), then $\mathcal{Q}_i = \emptyset$; or $\mathcal{S}_i \neq \emptyset$, then $\mathcal{Q}_i = \{\{j\}\}$ with $ij \in g$.

On the other hand, Definition 2.2 part (b) uses the following notion. For an expert $j \in M$, $\mathcal{M}_j$ is the set of expert sets $M'' \subseteq M$ that contain expert j and form a slight majority when adding their audiences of non-experts, i.e.

$$\sum_{k\in M''}(d_k+1)-\sum_{l\in M\setminus M''}(d_l+1)\in\{0,1,2\}. \tag{B.19}$$

Hence, there is a strong relation between the sets $\mathcal{S}_j$ and $\mathcal{M}_j$. To every set $S\in\mathcal{S}_j$ there corresponds one set $M''\in\mathcal{M}_j$ simply by $M''=S\cap M$, and equation B.18 above holds for the set S if and only if equation B.19 holds for the set $M''=S\cap M$.

Now, suppose a network is weakly balanced according to Definition A.1. We show weak balancedness according to Definition 2.2, which requires that for every expert $j\in M$, $\mathcal{M}_j\neq\emptyset$ implies that there is at least one element consisting of a weak majority of experts, i.e. $\exists M''\in\mathcal{M}_j$ such that $m''\geq\frac{m+1}{2}$. If for some expert $j\in M$, $\mathcal{M}_j=\emptyset$, then the condition cannot be violated for this particular expert. Consider any expert $j\in M$ with $\mathcal{M}_j\neq\emptyset$. Then $\mathcal{S}_j\neq\emptyset$, because $M'\in\mathcal{M}_j$ implies $M'\in\mathcal{S}_j$ and $\{\{j\}\}\in\mathcal{Q}_j\neq\emptyset$. By weak balancedness according to Definition A.1, $\exists S\in\mathcal{S}_j$ with $|M\cap S|>\frac{m}{2}$. We construct $M'':=S\cap M$, which satisfies $M''\in\mathcal{M}_j$ and $m''\geq\frac{m+1}{2}$.

Now, suppose a network is weakly balanced according to Definition 2.2. We show weak balancedness according to Definition A.1, which requires that for every voter $i\in V$ and for every $Q\in\mathcal{Q}_i$, there is a corresponding set of agents S with $Q\subseteq S\in\mathcal{S}_i$, which is better informed than the complementary set, i.e. $\prod_{j\in S}\frac{p_j}{1-p_j}\geq\prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$. For voters $i\in V$ with $\mathcal{Q}_i=\emptyset$, the condition cannot be violated for this particular voter i. Now, consider any expert $j\in M$ with $\mathcal{Q}_j\neq\emptyset$ and hence $\mathcal{S}_i\neq\emptyset$. Then $\mathcal{M}_j\neq\emptyset$, because $S\in\mathcal{S}_j$ implies $(S\cap M)\in\mathcal{M}_j$. By weak balancedness according to Definition 2.2, $\exists M''\in\mathcal{M}_j$ with $m''\geq\frac{m+1}{2}$. $M''\in\mathcal{M}_j$ means that $j\in M''$ and that M'' satisfies equation B.19 and thus also equation B.18 for $S=M''$. Hence, $|V^+(M'')|-|V^-(M'')|\in\{0,1,2\}$ such that $M''\in\mathcal{S}_j$. Moreover, $m''\geq\frac{m+1}{2}$ implies that $\prod_{j\in V\cap M''}\frac{p_j}{1-p_j}\geq\prod_{k\in V\setminus M''}\frac{p_k}{1-p_k}$ (because all experts $j\in M$ have equal signal precision p_j). This holds for any $Q\in\mathcal{Q}_i$ because all $Q\in\mathcal{Q}_i$ satisfy $Q\cap M''=\{j\}$ and non-experts do not affect the equations. Now, consider any non-expert $i\in N$ with $\mathcal{Q}_i\neq\emptyset$. Then $\mathcal{Q}_i=\{\{j\}\}\subseteq\mathcal{Q}_j$. Since for expert j linked to i there is a set $S=M''\in\mathcal{S}_j$ with $S\supseteq\{j\}$ with $\prod_{j\in S}\frac{p_j}{1-p_j}\geq\prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$, this also holds for non-expert i.

Strong balancedness implies weak balancedness. We show that a violation of weak balancedness implies a violation of strong balancedness.

Suppose weak balancedness is violated, i.e., there is a voter $i\in V$ and a set $Q\in\mathcal{Q}_i$, such that there is *no* corresponding set of agents S with $Q\subseteq S\in\mathcal{S}_i$, which is better informed than the complementary set, i.e., which is not fulfilling $\prod_{j\in S}\frac{p_j}{1-p_j}\geq\prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$. Hence, $\forall S\in\mathcal{S}_i$, we have $\prod_{j\in S}\frac{p_j}{1-p_j}<\prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$. ($\mathcal{S}_i\neq\emptyset$ because $\mathcal{Q}_i\neq\emptyset$ by assumption.) Then by strong balancedness, $|V^+(V\setminus S)|>|V^-(V\setminus S)|$, which implies $|V^+(S)|<|V^-(S)|$. However, this contradicts $S\in\mathcal{S}_i$, which requires that $|V^+(S)|-|V^-(S)|\in\{0,1,2\}$.

B.4 Simple Games: A Justification of ***Power***

Proposition A.2 can be interpreted in terms of expert power as defined in the class of *simple games* (cf., e.g., Roth, 1988). To see this, note that our model defines a non-cooperative game under incomplete information which is specified by an exogenous network g and by signal precisions p_j. To each of these games $\Gamma(g, p_1, ..., p_n)$ we will associate two cooperative games of the form (V, υ), with the characteristic function $\upsilon : 2^V \to \{0, 1\}$. In the first game (V, υ^*) a coalition S is winning, i.e., $\upsilon^*(S) = 1$, if and only if it has a larger expertise than the complementary set as quantified by the log-odds rule, i.e. $\sum_{j\in S} \log(\frac{p_j}{1-p_j}) > \sum_{k\in V\setminus S} \log(\frac{p_k}{1-p_k})$. (This is a so-called *weighted majority game* in which each voter j's weight is $\log(\frac{p_j}{1-p_j})$.) In the second game $(V, \hat{\upsilon})$ a coalition S is winning, i.e., $\hat{\upsilon}(S) = 1$, if and only if there are more believers than non-believers, i.e. $|V^+(S)| > |V^-(S)|$. This is a simple game which mimics the outcome of the sincere strategy profile in the game $\Gamma(g, p_1, ..., p_n)$. Indeed, if a set of voters S has received signal A^* and all others B^*, then under $\hat{\sigma}$ all $|V^+(S)|$ will vote for A, all $|V^-(S)|$ will vote for B, and all $|V^0(S)|$ will abstain.

In simple games, a player's *power* is measured by the Shapley value, which is then called the Shapley-Shubik index, or alternatively, with the Banzhaf index. Both indices take into account how often a player can "swing" a losing coalition into a winning coalition. In the simple game $(V, \hat{\upsilon})$ corresponding to Example 1, for instance, all five experts are equally powerful since the winning coalitions are those which have at least three expert members. This is also true in the other game (V, υ^*) that corresponds to Example 1 because all experts are equally well-informed. As a consequence, sincere voting is efficient in this example. The upcoming corollary of Proposition 2.2 shows that this relation between power and efficiency fully generalizes.

Definition B.1 (Power)**.** *For a weighted majority game (V, υ), define* ***power*** *of a player $i \in V$ as her Banzhaf index $\beta_i(\upsilon)$ or her Shapley-Shubik index $\phi_i(\upsilon)$. The (raw) Banzhaf index of a player $i \in V$ is the fraction of swings she has, i.e., $\beta_i(\upsilon) = \frac{1}{2^{n-1}} \sum_{S\subseteq V\setminus\{i\}}[\upsilon(S\cup\{i\})-\upsilon(S)]$; the Shapley-Shubik index of a player $i \in V$ is her marginal contribution averaged over all orderings of the players, which can be written as $\phi_i(\upsilon) = \sum_{S\subseteq V\setminus\{i\}} \frac{|S|!(|V|-|S|-1)!}{|V|!}[\upsilon(S\cup\{i\})-\upsilon(S)]$.*

In the game (V, υ^*) power only depends on the signal qualities. There $p_i > p_j$ implies that voter i is at least as powerful as expert j. In the game $(M, \hat{\upsilon})$, power is also monotonic in an agent's expertise p_i, in the sense that increasing a player's signal precision p_i cannot reduce her power. Similarly, in that game power is monotonic in a player's degree d_i in the sense that adding a new link ij to g cannot decrease the power of the agents i and j. However, a player's power in $(M, \hat{\upsilon})$ is not a simple function of her degree and her expertise, but depends on the network structure g as well as on the signal precisions. For every given example, it can be computed.

Proposition B.6. *Suppose there is no coalition $S \subset V$ with $\sum_{j\in S} \log(\frac{p_j}{1-p_j}) = \sum_{k\in V\setminus S} \log(\frac{p_k}{1-p_k})$.*[7]

[7] This assumption only rules out non-generic cases, in which after the realization of all signals still both alternatives are equally likely. In terms of simple games, the assumption means that the simple game (V, υ^*) is *strong*, i.e., for all coalitions $S \subset V$, $\upsilon^*(S) = 0$ implies that $\upsilon^*(V \setminus S) = 1$.

If the network g is strongly balanced, then each player's power is the same in the two corresponding games, i.e. $\forall j \in V,\ \ \phi_j(\hat{v}) = \phi_j(v^*)$, as well as $\beta_j(\hat{v}) = \beta_j(v^*)$. For the special case of homogenous signal quality among all experts, i.e. $p_j = p\ \ \forall j \in V$ with $p_j > 0.5$, strong balancedness means that each expert is equally powerful in $(V, \hat{v})$ and that all non-experts (with $p_i = 0.5$) have no power.

Proof. Recall that strong balancedness is defined as follows: $\forall S \subseteq V$, $\prod_{j\in S} \frac{p_j}{1-p_j} > \prod_{k\in V\setminus S} \frac{p_k}{1-p_k}$ implies $|V^+(S)| > |V^-(S)|$. By the definition of the games (V, v^*) and $(V, \hat{v})$, strong balancedness is equivalent to the following: $\forall S \subseteq V$, $v^*(S) = 1$ implies $\hat{v}(S) = 1$. Now, consider a set S such that $v^*(S) = 0$. By definition of v^*, we have either $\sum_{j\in S} \log(\frac{p_j}{1-p_j}) < \sum_{k\in V\setminus S} \log(\frac{p_k}{1-p_k})$ or $\sum_{j\in S} \log(\frac{p_j}{1-p_j}) = \sum_{k\in V\setminus S} \log(\frac{p_k}{1-p_k})$. The latter case is excluded by assumption. Hence, $v^*(S) = 0$ implies $v^*(V \setminus S) = 1$, which further implies by strong balancedness that $\hat{v}(V \setminus S) = 1$, which finally implies that $\hat{v}(S) = 0$. This shows for any set S that $\hat{v}(S) = 1$ if and only if $v^*(S) = 1$, which means that $\hat{v} = v^*$. As a consequence, the vectors of power coincide: $\phi(\hat{v}) = \phi(v^*)$, as well as $\beta(\hat{v}) = \beta(v^*)$.

We now turn to the special case of homogenous signal quality. Let $M \subseteq V$ denote the set of voters with an informative signal, which we call experts, i.e. $\forall j \in M$, we have $p_j = p > 0.5$. Since $\hat{v} = v^*$, it is sufficient to show that all experts $j \in M$ are equally powerful in (V, v^*) and that all non-experts $i \in V \setminus M$ (with $p_i = 0.5$) have power $\phi(v^*) = 0$, respectively, $\phi(v^*) = \beta(v^*) = 0$, in that game (V, v^*).

A non-expert $i \in V \setminus M$ contributes $\log(\frac{0.5}{1-0.5}) = 0$ to each coalition S such that he is a so-called dummy player: $\forall S \subseteq V \setminus \{i\}$ we have $v(S \cup \{i\}) = v(S)$. By definition of the Shapley-Shubik index and the Banzhaf index, non-expert i's power is thus zero: $\beta_i(v) = 0$, respectively $\phi_i(v^*) = 0$.

All experts $j \in M$ contribute $\log(\frac{p}{1-p}) > 0$ to each coalition S such that they are symmetric in the game (V, v^*).[8] Consequently, all experts are equally powerful.

□

The proposition gives an interpretation to Proposition 2.2 by showing that strong balancedness means that there are the same winning coalitions in the two corresponding games. When signal precisions are homogeneous, all experts are equally powerful in (V, v^*) such that it is intuitive that equal power of experts in $(V, \hat{v})$ means efficiency of $\hat{\sigma}$. This can be illustrated with Example 1, in which each expert is indeed equally powerful in the game $(V, \hat{v})$ since the winning coalitions are those which have at least three members.

To illustrate a violation of strong balancedness, we consider an extreme case, in which there is a *dictator*, i.e., a player j who has a swing in every coalition $S \subseteq V \setminus \{j\}$. A dictator has the maximal Banzhaf index and the maximal Shapley-Shubik index of one. Any player following the dictator's message is "cursed" in the sense that if the own vote is decisive under $\hat{\sigma}$, then the opposite of the message is more likely to be correct. An example illustrating this

[8] A pair of players $i, j \in V$ is called symmetric if $\forall S \subseteq M \setminus \{i, j\}$ we have $v(S \cup \{i\}) = v(S \cup \{j\})$. If two players i and j are symmetric, then they always have the same Banzhaf index $\beta_i(v) = \beta_j(v)$, respectively the same Shapley-Shubik index $\phi_i(v) = \phi_j(v)$, by definition of the two indices.

effect is given by the weighted majority game $(V, \hat{v})$ corresponding to Example 2, the star network, in which expert 1 has dictatorial power.[9]

B.5 Complete Proof of Proposition A.3

We show existence of inefficient strategy profiles with the network introduced in Example 3 and extensions of it. For any $t = 1, 2, ...$ we consider a network with two experts of degree $2t$, $1 + 2t$ experts of degree zero and $4t$ non-experts of degree one. For $t = 1$ this is exactly the network depicted in Figure 2. All experts have signal quality $p_j = p > 0.5$, all non-experts signal quality $p_i = 0.5$. For any $t = 1, 2, ...$, denote the corresponding game by Γ^t and the sincere strategy profile in that game by $\hat{\sigma}^t$.

Under $\hat{\sigma}^t$, $3 + 6t$ agents participate in the vote and a majority is reached with at least $2 + 3t$ votes. If the two senders receive the same signal, say A^*, then A is the outcome since the two senders induce $2 * (1 + 2t) \geq 2 + 3t$ A-votes. If both senders receive different signals, A^* and B^*, then A wins if and only if A receives $k \geq 1 + t$ votes of the $1 + 2t$ experts with degree zero. Supposing that A is the true state, the probability that the outcome is A provides the general probability that the outcome coincides with the true state since $\hat{\sigma}^t$ treats A and B interchangeably. Thus, under $\hat{\sigma}^t$ the probability that the outcome coincides with the true state is

$$EU(\hat{\sigma}^t) = p^2 * 1 + 2p(1-p) \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + (1-p)^2 * 0. \tag{A.6}$$

Inefficiency. We establish inefficiency of $\hat{\sigma}^t$ for any t and also in the limit. (Recall that a strategy profile is efficient if and only if for any draw of nature it selects the outcome that maximizes the probability to match the true state.) Consider the draw of nature in which both senders receive signal A^* and all other experts receive signal B^*. An efficient strategy profile would implement (the majority signal) B, but $\hat{\sigma}^t$ leads to A.

For an efficient strategy profile σ^t the probability that the outcome coincides with the true state is below one for finite t, but converges to one for growing t, i.e. $\lim_{t\to\infty} EU(\sigma^t) = 1$ when σ^t efficient. Under $\hat{\sigma}^t$, when both senders happen to receive the incorrect signal, then the outcome does not coincide with the true state. Thus, the probability of implementing the incorrect outcome under $\hat{\sigma}^t$ is at least $(1-p)^2$, which is independent of t. Hence, $\lim_{t\to\infty} EU(\hat{\sigma}^t) \leq 1 - (1-p)^2 < 1$, i.e., inefficiency does not vanish for growing t.

Now, we establish that $\hat{\sigma}^t$ is an equilibrium for any t. We show first that there is no profitable deviation that occurs on the voting stage only. Then we show that there is no profitable deviation that affects both stages voting and communication.

[9] The simple games corresponding to Examples 1 and 2 are extreme cases with minimal, respectively maximal, inequality of expert power.

Deviations on the voting stage only. Consider a voter $i \in V$ who considers to deviate from $\hat{\sigma}^t$ by changing his voting strategy v_i. This can be a non-expert who does not follow the received message or an expert who does not vote the received signal, but chooses some different strategy instead.

Suppose one sender (i.e., a voter with $p_j = p > 0.5$ and $d_j = 2t$) receives signal A^* and the other sender receives signal B^*. Then A receives more votes than B under $\hat{\sigma}^t$ if and only if more experts with degree zero (i.e., voters with $p_j = p > 0.5$ and $d_j = 0$) have received signal A^*. Hence, when the two senders have not received the same signal, then $\hat{\sigma}^t$ always implements the majority signal and hence induces the outcome that is more likely to be true. Hence, if there is a beneficial deviation, then it must also change outcomes in which both senders have received the same signal.

Suppose that both senders have received the same signal, say A^*. Then the number of A-votes under $\hat{\sigma}^t$ is at least $2 + 4t$ (since two senders, and $2 * 2t$ non-experts vote for A) and the number of B-votes is hence at most $3 + 6t - (2 + 4t) = 1 + 2t$. The number of A-votes thus exceeds the number of B-votes by at least $2 + 4t - (1 + 2t) = 1 + 2t \geq 3$ votes. Hence, a single agent who changes her vote cannot affect the outcome if the two senders have received the same signal.

Taken together a deviation that only changes one vote is neither beneficial if both senders have received the same signal nor if they have received different signals. This precludes deviation incentives of non-experts, of experts with degree zero, as well as of senders who consider to deviate in their voting behavior only, i.e., all deviations that happen on the voting stage only. We now turn to deviations that also affect the communication stage, i.e., which involve a sender who does not truthfully transmit her signal, and show that any of those is neither beneficial.[10]

Deviations on both stages. Consider a sender $j \in V$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals.[11] To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A^* because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to the following nine: $(m_j(A^*), v_j(A^*)) \in \{(A, A), (A, B), (A, \emptyset), (B, A), (B, B), (B, \emptyset), (\emptyset, A), (\emptyset, B), (\emptyset, \emptyset)\}$. The first strategy (A, A) is sincere and hence not a deviation. The strategies (A, B) and $(A, \emptyset)$ only involve deviations on the voting stage and are hence not beneficial by the paragraph above. This leads to the following six remaining deviations $\tilde{\sigma}$ and their corresponding expected utilities $EU(\tilde{\sigma}^t)$:[12]

[10] For large t this is simple to show. In the case in which the deviating agent receives the correct signal, say A^*, and the other sender receives the incorrect signal, the probability that the outcome is A approaches zero for growing t. Hence, the expected utility of any such deviation is bounded from above by $\lim_{t\to\infty} EU(\tilde{\sigma}^t) \leq 1 - p(1-p) < 1 - (1-p)^2 = \lim_{t\to\infty} EU(\hat{\sigma}^t)$.

[11] In general, voters with positive degree $d_i > 0$ have more pure strategies. In this example, the senders are linked to non-experts (i.e voters i with $p_i = 0.5$) who are assumed by convention not to send a message under $\hat{\sigma}^t$. Since a message of an uninformed voter is meaningless, a change of convention would not affect the result.

[12] Deviations that involve to vote and/or communicate an alternative unconditionally, i.e., independent of the signal, need not be considered here because of the symmetry between the alternatives. Indeed, if it is

1. Sender j sends the opposite message and votes the signal.

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=t}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) + (1-p)^2 \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p) \quad \text{(A.7)}$$

2. Sender j sends the opposite message and votes the opposite.

$$EU(\tilde{\sigma}^t) = [p^2 + (1-p)^2] \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) \quad \text{(A.8)}$$

3. Sender j sends the opposite message and abstains.

$$EU(\tilde{\sigma}^t) = p^2 \left[\frac{1}{2} \binom{2t+1}{t} p^t (1-p)^{t+1} + \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right]$$
$$+ p(1-p) + (1-p)^2 \left[\frac{1}{2} \binom{2t+1}{t+1} p^{t+1} (1-p)^t + \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right],$$

which is equation A.9

4. Sender j sends the empty message and votes the signal.

$$EU(\tilde{\sigma}^t) = p^2 + p(1-p)p^{2t+1} + p(1-p) \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \quad \text{(A.10)}$$

5. Sender j sends the empty message and votes the opposite.

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) + (1-p)^2 p^{2t+1} \quad \text{(A.11)}$$

6. Sender j sends the empty message and abstains.

$$EU(\tilde{\sigma}^t) = p^2[1 - \frac{1}{2}(1-p)^{2t+1}] + p(1-p)\frac{1}{2}p^{2t+1} + p(1-p)[1 - \frac{1}{2}(1-p)^{2t+1}] + (1-p)^2 \frac{1}{2} p^{2t+1} \quad \text{(A.12)}$$

The derivation of the expressions (A.7)-(A.12) is shown below. We can then compare the expected utility $EU(\tilde{\sigma}^t)$ of each deviation, which is given by (A.7)-(A.12), with the expected utility of the sincere strategy profile $EU(\hat{\sigma}^t)$, which is given by (A.6).

Consider, for instance, the fifth deviation: Sender j sends the empty message and votes the opposite of the signal. There are $3 + 4t$ votes and $2 + 2t$ is a majority. Denote by (s_j, s_k) the signals of the two senders. There are four possibilities.

- (A^*, A^*): A wins if there are at least $2 + 2t - (1 + 2t) = 1$ A^*-signals among the experts of degree zero.

beneficial to vote B after receiving A^*, then it is also beneficial to vote A after receiving B^*, which is to vote the opposite of the signal. Similarly, there is no need to consider strategies that involve the empty message and/or to abstain only after one of the two signals. Indeed, if it is beneficial e.g. to abstain after having received signal A^*, then it is also beneficial to abstain after having received signal B^*, which is to abstain unconditionally. Hence, if none of the six symmetric deviations is an improvement over $\hat{\sigma}^t$, then neither is a deviation that treats the alternatives A and B asymmetrically.

- (A^*, B^*): A never wins since B receives at least $2+2t$ votes.
- (B^*, A^*): A wins since it receives at least $2+2t$ votes.
- (B^*, B^*): A wins if there are at least $2+2t-1 = 2t+1$ A^*-signals among the experts of degree zero, i.e., all of them have signal A^*.

We now show that this deviation is not beneficial by considering the change in expert j's expected utility (which is the expected utility of every agent). Supposing that the true state is A, the expected utility is the likelihood that A is indeed implemented. Hence,

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) * 0 + p(1-p) * 1 + (1-p)^2 p^{2t+1},$$

which directly simplifies to (A.11).

For the upcoming simplifications we use the following two properties:

1. $\sum_{k=0}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} = 1$ and
2. $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$ for any $k = 0, ..., 2t+1$.

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$. Then

$$\Delta = p^2 \left[1 - \sum_{k=1}^{2t+1} (...)\right] + p(1-p)\left[2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} - 1\right] - (1-p)^2 p^{2t+1}$$

$$\begin{aligned}
\Delta &= p^2 \left[\sum_{k=0}^{2t+1} (...) - \sum_{k=1}^{2t+1} (...)\right] + p(1-p)\left[2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} - 1\right] \\
&\quad -(1-p)^2 p^{2t+1} \\
\Delta &= p^2 (1-p)^{2t+1} + p(1-p) \underbrace{\left[\sum_{k=t+1}^{2t+1} (...) - \sum_{k=0}^{2t+1} (...)\right]}_{=-\sum_{k=0}^{t}(...)} + \underbrace{p(1-p) \sum_{k=t+1}^{2t+1} (...) - (1-p)^2 p^{2t+1}}_{\geq p(1-p)\sum_{k=t+1}^{2t}(...)}
\end{aligned}$$

To simplify the last part of the equation notice the following:

- First, $\sum_{k=t+1}^{2t+1} (p^k (1-p)^{2t+1-k} = \sum_{k=t}^{2t} (p^k (1-p)^{2t+1-k} + \binom{2t+1}{2t+1} p^{2t+1} (1-p)^0$.
- Second, $\binom{2t+1}{2t+1} p^{2t+1} (1-p)^0 = p^{2t+1}$.
- Third, $p(1-p)p^{2t+1} - (1-p)^2 p^{2t+1} = [p(1-p) - (1-p^2)]p^{2t+1} \geq 0$.

Thus,

$$\begin{aligned}
\Delta &\geq p^2(1-p)^{2t+1} - p(1-p)\sum_{k=0}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)\sum_{k=t+1}^{2t}(...) \\
\Delta &\geq \underbrace{p^2(1-p)^{2t+1} - p(1-p)\binom{2t+1}{0}p^0(1-p)^{2t+1}}_{\geq 0} - p(1-p)\sum_{k=1}^{t}(...) + p(1-p)\sum_{k=t+1}^{2t}(...) \\
\Delta &\geq \underbrace{p(1-p)}_{\geq 0}\left[\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} - \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}\right]
\end{aligned}$$

Hence, $\Delta \geq 0$ if

$$\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} \geq \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}. \tag{A.13}$$

To show that inequality A.13 holds, we substitute k in the first sum by $l \equiv 2t+1-k$ and consistently sum over $l = 1, ..., t$ (instead over $k = t+1, ..., 2t$). Moreover, we use $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$.

$$\begin{aligned}
\sum_{l=1}^{t}\binom{2t+1}{l}p^{2t+1-l}(1-p)^l - \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} &\geq 0 \\
\sum_{l=1}^{t}\binom{2t+1}{l}\left(p^{2t+1-l}(1-p)^l - p^l(1-p)^{2t+1-l}\right) &\geq 0.
\end{aligned}$$

For every $l = 1, ..., t$, we have $2t+1-l > l$. This implies for the expression in brackets that the first product $(p^{2t+1-l}(1-p)^l)$ is larger than the second product $(p^l(1-p)^{2t+1-l})$. Hence, the inequality above holds, which implies inequality A.13. Thus, $EU(\hat{\sigma}^t) \geq EU(\tilde{\sigma}^t)$ and hence this deviation $\tilde{\sigma}^t$ is not beneficial.

Using the same techniques as for the deviation above, we will show for the other five deviations $\tilde{\sigma}^t$ that $EU(\tilde{\sigma}^t) \leq EU(\hat{\sigma}^t)$.

1. Sender j sends the opposite message and votes the signal. There are $3+6t$ votes and $2+3t$ is a majority.

   - (A^*, A^*): A wins if there are at least $2+3t-(2+2t) = t$ A^*-signals among the experts of degree zero.
   - (A^*, B^*): A never wins since $1+1+2t < 2+3t$.
   - (B^*, A^*): A wins since $1+4t \geq 2+3t$.
   - (B^*, B^*): A wins if there are at least $2+3t-2t = 2+t$ A^*-signals among the experts of degree zero.

$$EU(\tilde{\sigma}^t) = p^2\sum_{k=t}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)*0 + p(1-p)*1 + (1-p)^2\sum_{k=t+2}^{2t+1}\binom{2t+1}{k}p^k(1-p)$$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[1 - \sum_{k=t}^{2t+1}(...)] + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1\right] - (1-p)^2\sum_{k=t+2}^{2t+1}(...)$$

$$\begin{aligned}
\Delta &= p^2\left[\sum_{k=0}^{2t+1}(...) - \sum_{k=t}^{2t+1}(...)\right] + p(1-p)\left[\sum_{k=t+1}^{2t+1}(...) - \sum_{k=0}^{2t+1}(...)\right] + \\
&\quad \underbrace{p(1-p)\sum_{k=t+2}^{2t+1}(...) - (1-p)^2\sum_{k=t+2}^{2t+1}(...)}_{\geq 0} + p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t \\
\Delta &\geq \underbrace{p^2\sum_{k=0}^{t-1}(...) - p(1-p)\sum_{k=0}^{t}(...)}_{\geq -p(1-p)\binom{2t+1}{t}p^t(1-p)^{t+1}} + p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t \\
\Delta &\geq p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t - p(1-p)\binom{2t+1}{t}p^t(1-p)^{t+1}
\end{aligned}$$

Hence $\Delta \geq 0$ if

$$\begin{aligned}
\binom{2t+1}{t+1}p^{t+1}(1-p)^t &\geq \binom{2t+1}{t}p^t(1-p)^{t+1} \\
p^{t+1}(1-p)^t &\geq p^t(1-p)^{t+1} \\
p &\geq 1-p,
\end{aligned}$$

which is true.

2. Sender j sends the opposite message and votes the opposite of the signal. There are $3+6t$ votes and $2+3t$ is a majority.

   - (A^*, A^*): A wins if there are at least $2+3t-(1+2t) = 1+t$ A^*-signals among the experts of degree zero.
   - (A^*, B^*): A never wins since $1+2t < 2+3t$.
   - (B^*, A^*): A wins since $2+4t \geq 2+3t$.
   - (B^*, B^*): A wins if there are at least $2+3t-1+2t = 1+t$ A^*-signals among the experts of degree zero.

$$EU(\tilde{\sigma}^t) = p^2\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)*0 + p(1-p)*1 + (1-p)^2\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)$$

$$EU(\tilde{\sigma}^t) = [p^2 + (1-p)^2]\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)$$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[1 - \sum_{k=t+1}^{2t+1}(...)] - (1-p)^2 \sum_{k=t+1}^{2t+1}(...) + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1\right]$$

$$\Delta = [p(1-p) - (1-p)^2]\sum_{k=t+1}^{2t+1}(...) + [p^2 - p(1-p)[1 - \sum_{k=t+1}^{2t+1}(...)] \geq 0,$$

which is positive, since both summands are positive.

3. Sender j sends the opposite message and abstains. There are $2+6t$ votes and $1+3t$ is a tie.

   - (A^*, A^*): there is a tie if there are $1+3t-(1+2t) = t$ A^*-signals among the experts of degree zero. For more, A wins.
   - (A^*, B^*): A never wins since $1+2t < 1+3t$.
   - (B^*, A^*): A wins since $1+4t > 1+3t$.
   - (B^*, B^*): there is a tie if there are $1+3t-2t = 1+t$ A^*-signals among the experts of degree zero. If there are $k \geq= 2+t$ A^*-signals A wins.

$$EU(\tilde{\sigma}^t) = p^2\left[\frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1} + \sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}\right] + p(1-p)*0$$

$$+p(1-p)*1 + (1-p)^2\left[\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t + \sum_{k=t+2}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}\right]$$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[1 - [\frac{1}{2}...]] + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1\right] - (1-p)^2[\frac{1}{2}...]$$

$$
\begin{aligned}
\Delta &= p^2\left[\sum_{k=0}^{t}(...) - \frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1}\right] - p(1-p)\sum_{k=0}^{t}(...) + p(1-p)\sum_{k=t+1}^{2t+1}(...) - \\
&\quad (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t - (1-p)^2\sum_{k=t+2}^{2t+1}(...) \\
\Delta &= \underbrace{p^2\sum_{k=0}^{t}(...) - p(1-p)\sum_{k=0}^{t}(...)}_{\geq 0} + \underbrace{p(1-p)\sum_{k=t+1}^{2t+1}(...) - (1-p)^2\sum_{k=t+2}^{2t+1}(...)}_{\geq p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t} \\
&\quad -p^2\frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1} - (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t \\
\Delta &\geq \underbrace{p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t - (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t}_{\geq \frac{1}{2}p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t} - p^2\frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1} \\
\Delta &\geq \frac{1}{2}p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t - \frac{1}{2}p^2\binom{2t+1}{t}p^t(1-p)^{t+1}
\end{aligned}
$$

Hence $\Delta \geq 0$ if

$$
\begin{aligned}
p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t &\geq p^2\binom{2t+1}{t}p^t(1-p)^{t+1} \\
p(1-p)p^{t+1}(1-p)^t &\geq p^2p^t(1-p)^{t+1} \\
p^{t+2}(1-p)^{t+1} &\geq p^{t+2}(1-p)^{t+1},
\end{aligned}
$$

which is true.

4. Sender j sends the empty message and votes the signal. There are $3 + 4t$ votes and $2+2t$ is a majority. If both senders receive the same signal, say A^*, A wins since there are at least $2+2t$ A-votes. Hence, the outcome is not different from $\hat{\sigma}^t$. If both senders receive different signals, then the outcome under $\hat{\sigma}^t$ is optimal such that there cannot be a beneficial deviation.

5. Sender j sends the empty message and votes the opposite of the signal. It has been already shown above that this deviation is not beneficial.

6. Sender j sends the empty message and abstains. Then there are $2+4t$ votes and $1+2t$ is just half of all votes.

   - (A^*, A^*): there is a tie if there are $1 + 2t - (1 + 2t) = 0$ A^*-signals among the experts of degree zero. Otherwise, A wins.
   - (A^*, B^*): there is a tie if there are $1+2t-0 = 1+2t$ A^*-signals among the experts of degree zero. Otherwise, B wins.
   - (B^*, A^*): there is a tie if there are $1 + 2t - (1 + 2t) = 0$ A^*-signals among the experts of degree zero. Otherwise A wins.

- (B^*, B^*): there is a tie if there are $1+2t-0 = 1+2t$ A^*-signals among the experts of degree zero. Otherwise, B wins.

$EU(\tilde{\sigma}^t) = p^2[1-\frac{1}{2}(1-p)^{2t+1}]+p(1-p)\frac{1}{2}p^{2t+1}+p(1-p)[1-\frac{1}{2}(1-p)^{2t+1}]+(1-p)^2\frac{1}{2}p^{2t+1}$
Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[\frac{1}{2}(1-p)^{2t+1}]+p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1 + \frac{1}{2}p^{2t+1} - \frac{1}{2}(1-p)^{2t+1}\right] - (1-p)^2\frac{1}{2}p^{2t+1}$$

$$\begin{aligned}
\Delta &= \underbrace{(p^2 - p(1-p))\frac{1}{2}(1-p)^{2t+1}}_{\geq 0} + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1 + \frac{1}{2}p^{2t+1}\right] - (1-p)^2\frac{1}{2}p^{2t+1} \\
\Delta &\geq p(1-p)\left[\sum_{k=t+1}^{2t+1}(...) + \sum_{k=t+1}^{2t+1}(...) - \sum_{k=0}^{2t+1}(...) + \frac{1}{2}p^{2t+1}\right] - (1-p)^2\frac{1}{2}p^{2t+1} \\
\Delta &\geq \underbrace{p(1-p)\left[\sum_{k=t+1}^{2t+1}(...) - \sum_{l=0}^{t}(...)\right]}_{\geq 0} + \underbrace{(p(1-p) - (1-p)^2)}_{\geq 0}\frac{1}{2}p^{2t+1} \\
\Delta &\geq p(1-p)\left[\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^l(1-p)^{2t+1-k} - \sum_{l=0}^{t}\binom{2t+1}{l}p^l(1-p)^{2t+1-l}\right] \\
\Delta &\geq p(1-p)\left[\sum_{l=0}^{t}\binom{2t+1}{l}p^{2t+1-l}(1-p)^l - \sum_{l=0}^{t}\binom{2t+1}{l}p^l(1-p)^{2t+1-l}\right] \\
\Delta &\geq \underbrace{p(1-p)\left[\sum_{l=0}^{t}\binom{2t+1}{l}\left(p^{2t+1-l}(1-p)^l - p^l(1-p)^{2t+1-l}\right)\right]}_{\geq 0}
\end{aligned}$$

For every $l = 1, ..., t$, we have $2t+1-l > l$. This implies for the expression in brackets that the first product $(p^{2t+1-l}(1-p)^l)$ is larger than the second product $(p^l(1-p)^{2t+1-l})$. Thus, the expression in brackets is positive.

$\square$

C Supplementary Experimental Appendix

C.1 Additional Tables

Table C.1.1. Individual sincere behavior of experts by position

	never	always	strongly balanced	weakly balanced	star
empty: *pos. 1-5*	1.9%	59.1%	0.161	0.450	0.000
strongly balanced: *pos. 5*	12.6%	70.5%		0.353	0.715
weakly balanced: *pos. 3-5*	2.9%	61.9%			0.047
star: *pos. 2-5*	1.9%	52.4%			
strongly balanced: *pos. 1-4*	4.8%	49.5%		0.039	0.003
weakly balanced: *pos. 1-2*	12.5%	43.3%			0.085
star: *pos. 1*	31.6%	43.2%			

Table C.1.1: Individual behavior of experts by position: for each individual in each network position (she is in) there is a variable capturing the frequency of sincere actions. The network positions refer to the upper panel of Figure 3. The first block compares experts who are not senders across treatments. The second block compares experts who are senders across treatments. Experts are at most ten times in each position. Column 2 and 3 report the fraction of participants who *never* respectively *always* chose the sincere strategy in the given position. Columns 4-6 of the table show the p-values of Wilcoxon matched-pairs signed-ranks test.

Table C.1.2. Efficiency, expected payoff EP and success

treatment	win	lose	EP	success
empty	74.3%	11.9%	64.2	62.9%
strongly balanced	77.6%	13.3%	64.2	65.7%
weakly balanced	74.3%	14.8%	63.7	60.5%
star ($N = 210$)	68.6%	18.6%	60.8	60.5%
star with majority signal for position 1 ($N = 132$)	81.1%	9.1%	65.1	62.1%
star with minority signal for position 1 ($N = 78$)	47.4%	34.6%	53.5	57.7%
Total	73.7%	14.6%	63.2	62.4%

Table C.1.2: Efficiency, expected payoff EP, and success. 'Win' (respectively 'lose') means that the outcome of voting is the majority signal (respectively the minority signal); in addition to the displayed categories 'win' and 'lose' the outcome can be a tie. EP can be interpreted as the likelihood in percent that the group decision matches the true state. 'Success' is the fraction of group decisions which were actually correct. If we consider reasonable values of EP to lie between the EP of a dictator who is chosen from the set of experts and the EP of an efficient strategy profile, then the reasonable range is $[60.0, 68.3]$.

Table C.1.3. Dependent variable: Expected payoff EP

	OLS 1		OLS 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
empty			-0.231	(0.644)
strongly balanced	-0.110	(1.489)	-0.341	(1.196)
weakly balanced	0.231	(0.644)		
star	-1.356*	(0.646)	-1.587*	(0.678)
uniform signal	33.309***	(0.506)	33.309***	(0.506)
almost uniform signal	18.202***	(1.680)	18.202***	(1.680)
Intercept	54.272***	(0.567)	54.503***	(0.424)
N	840		840	
R^2	0.534		0.534	
p-value F-test	0.000		0.000	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table C.1.3: Estimation results: OLS with expected payoff EP as dependent variable. Robust standard errors in parentheses adjusted for sessions. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table C.1.4. Avoidability of inefficiency

	vote difference	"wrong" experts	"wrong" non-experts	preventable
empty ($N = 54$)	0.74	2.26	1.11	51.9%
strongly balanced ($N = 47$)	1.09	2.19	2.04	72.3%
weakly balanced ($N = 54$)	1.24	2.22	1.87	59.3%
star ($N = 66$)	1.41	2.15	2.05	60.6%
Total ($N = 221$)	1.14	2.20	1.77	60.6%

Table C.1.4: Avoidability of inefficiency. The variable 'vote difference' refers to the absolute difference of the number of votes. A vote difference of, e.g., 2 means that the minority signal has received two more votes than the majority signal; and a vote difference of 0 means that a tie has occurred. The label "wrong" refers to an agent who voted for the minority signal. The table reports the mean of these variables over all inefficient cases, i.e., for all groups where the majority signal did not receive a majority of votes. Column 5 'preventable' reports the fraction of groups that would have avoided an inefficient outcome if all "wrong" non-experts abstained. On average the vote difference is 0.68, reflecting that most inefficient outcomes are close calls such as ties (where the vote difference is zero) or wins of the minority signal by one vote (where the vote difference is one). Comparing this number to the number of experts and the number of non-experts who voted for the minority signal indicates who could have prevented the inefficiency. In the non-empty networks, there are on average roughly two non-experts who voted for the minority signal. If they abstained, the efficient outcome would have been reached in most of the cases.

C.2 Instructions

The original instructions are written in German and can be requested from the authors. On the next pages we provide an English version which is a sentence-by-sentence translation of the original instructions. The instructions are followed by the questions of comprehension.

Welcome to today's experiment!

Please note that **no communication** is allowed from now on and during the whole experiment. If you have a question please raise your hand from the cabin, one of the experimenters will then come to you. The use of cell phones, smart phones, tablets, or similar devices is prohibited during the entire experiment. Please note that a violation of this rule leads to exclusion from the experiment and from any payments.

All decisions are taken anonymously, i.e. none of the other participants comes to know the identity of the others. The payoff is also conducted anonymously at the end of the experiment.

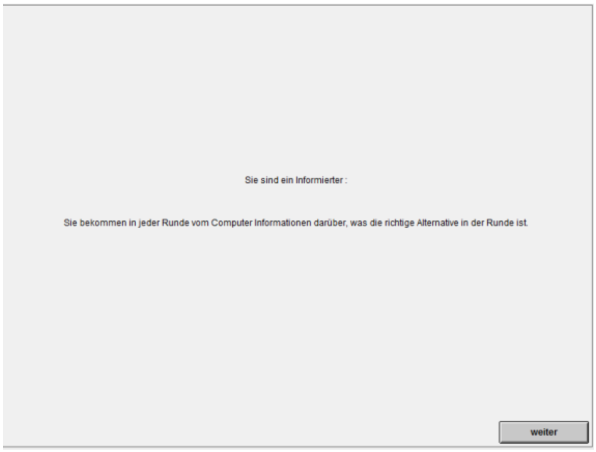
Instructions

In this experiment you will choose along with your group one out of two alternatives whereupon just one alternative is correct and the other is wrong. Only the correct alternative leads to a positive payoff for each member of the group. Some members of the group will receive information about the correct alternative. This information is accurate in 60 out of 100 cases. The group decides by voting which alternative will be implemented. The group is furthermore arranged in a communication network. Certain members of the group can – depending on the network structure – transmit a message to other members before the group ballots for the alternatives.

The sequence of each individual round consists of the following 4 parts.

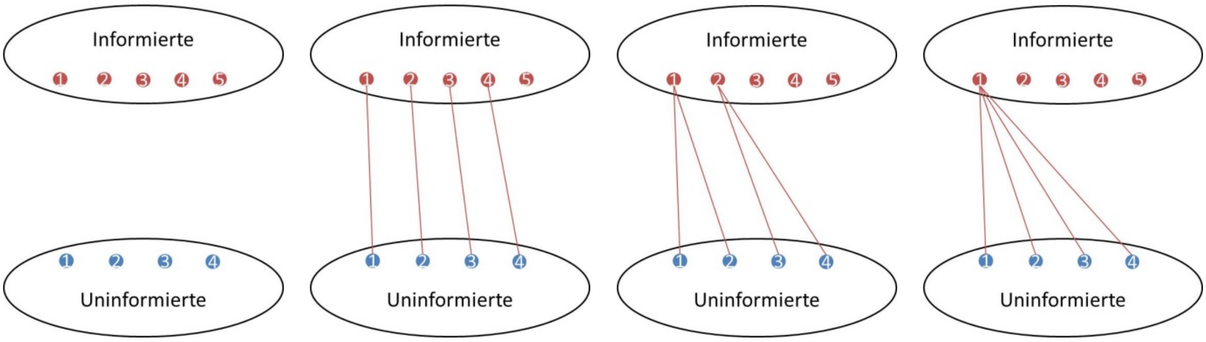
1. Information

You will receive the role of an Informed or an Uninformed at random (and you will keep it during the entire experiment). There are two alternatives: alternative "circle" and alternative "triangle". At the beginning of each round one of the two alternatives will be assigned at random and with equal likelihood as the correct alternative. The "Informed" receive information about the correct alternative which is accurate in 60 out of 100 cases. (The Informed will not necessarily all receive the same information). The "Uninformed" will not receive any information about what the correct alternative is.

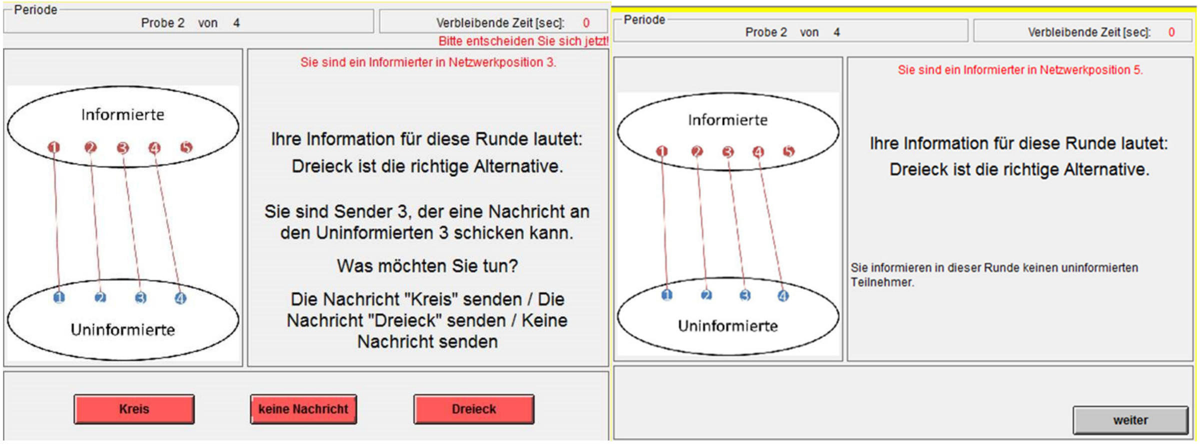


2. Communication

You will randomly be divided into groups of 9 members. A group is composed of 5 Informed and 4 Uninformed. All group members are arranged in a communication network. At the beginning of a round you get to know the network structure and your position in the network. You can see the possible networks pictured in the figure below.

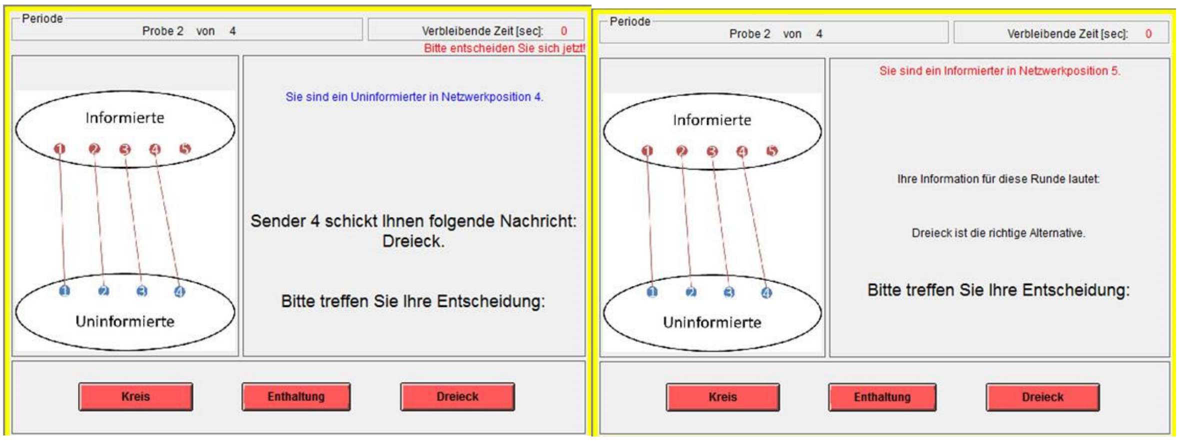


5 Informed receive in randomized arrangement the positions *Above 1 to 5* in the network. 4 Uninformed receive in randomized arrangement the positions *Below 1 to 4* in the network. Everyone knows therefore that someone with an upper position is an Informed and that someone with a lower position is an Uninformed. The network structure reveals who can communicate with whom. The Uninformed can be recipients but not senders of a message. The Informed who are in the position of a sender send either the message “circle” or the message “triangle” or they don’t send any message to their recipient(s). Each sender can send exactly one message to all of its (his/her) recipients. Not every Informed is necessarily a sender. This depends on the network structure and the network position. The connecting lines between upper and lower positions in the network display who can send a message to whom.



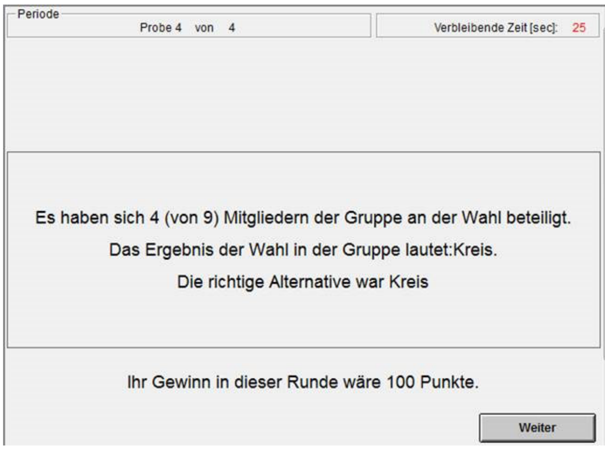
3. Voting

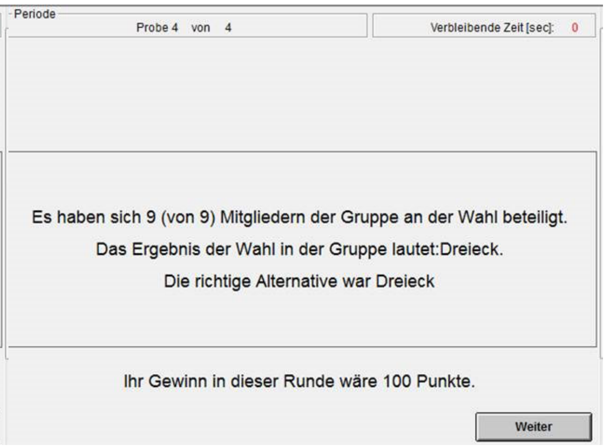
You can decide to vote for “circle,” to abstain from voting, or to vote for “triangle.” The voting result in the group is the alternative (circle or triangle) with the most votes. In case of a tie the computer will pick one of the two alternatives at random and with the same probability.



4. Outcome

At the end of the round you will get to know the voting outcome as well as the right alternative. If they match, e.g. the voting outcome is triangle and the right alternative is triangle, you will receive 100 points. Otherwise you will not receive any points. At the end of 40 rounds 3 rounds will be drawn randomly, which are then relevant for the payoffs. The rate of exchange between points and Euro is the following: 20 points correspond to 1 Euro. You will receive 5 Euro additionally for your participation in the experiment.





Procedure of the experiment

40 rounds will be played in total. The composition of the group changes from round to round. The network structure changes every 5 rounds. There will be a short questionnaire subsequent to the 40 rounds of the experiment. Prior to the 40 rounds of the experiment 4 sample rounds are played. These are not payoff-relevant. (In each sample round a different network is introduced.)

Summary of the procedure of the experiment:

1. Reading of the instructions
2. Questions of comprehension concerning the instructions
3. 4 sample rounds
4. 40 EXPERIMENTAL ROUNDS
5. Questionnaire
6. Payoffs

If you have a question, please raise your hand from the cabin, we will then come to you.

Comprehension questions

1. Which of the following statements is correct? (Please checkmark)
    a. The role of the Informed/Uninformed changes from round to round.
    b. The group affiliation changes from round to round.
    c. The network changes from round to round.

2. Which of the following statements is correct? (Please checkmark)
    a. In each round either the alternative „circle" or the alternative „triangle" is correct, namely with a probability of 50% no matter which alternative has been most frequently correct in the previous rounds.
    b. If „triangle" was 7 times correct in the previous 10 rounds and „circle" only 3 times, then in the current round it is more likely that „circle" is correct instead of „triangle".
    c. If „circle" was 7 times correct in the previous 10 rounds and „triangle" only 3 times, then in the current round it is more likely that „circle" is correct instead of „triangle".

3. Which of the following statements is correct? (Please checkmark)
    a. The „Informed" in the group know for sure which alternative is correct.
    b. All „Informed" in the group share the same opinion about what the correct alternative is.
    c. Each „Informed" in the group receives some information about which alternative is correct and this information is accurate in 60 out of 100 cases.

4. Which of the following statements is correct? (Please checkmark)
    a. Each „Informed" is a sender.
    b. Each sender is an „Informed."
    c. A sender can be an „Informed" or an "Uninformed."

5. Which of the following statements is correct? (Please checkmark)
    a. If the correct alternative is „circle" and you vote for circle, you will always receive 100 points.
    b. If the correct alternative is „circle" and a majority of the participants vote for circle, you will receive 100 points.
    c. If the correct alternative is „circle" and a majority of the participants vote for triangle, you will receive 100 points.

C.3 Partisans and Study II

In an earlier working paper version that is permanently available online (Buechel and Mechtenberg, 2017), we have relaxed the assumption that all agents have the same preferences, i.e., that they all want the policy to match the state of the world. There, we introduce agents who try to induce a specific policy regardless of the state of the world, e.g., due to the expectation of personal perquisites. We call these trolls A-partisans or B-partisans according to their preferred policy. When the number of A-partisans equals the number of B-partisans our three results, Proposition 2.1-2.3, extend to this extended model set-up, as we prove there.[13] Hence, the model with partisans leads to fully analogous predictions.

We have tested this extended model in a second experiment, to which we refer to as Study II. In the experimental implementation voter groups – i.e., subject groups interacting in one network – consist of three experts, four computerized partisans, and four non-experts. The four partisans divide into two A-partisans who always communicate and vote A and two B-partisans who always communicate and vote B.

In the experimental design of Study II, we again implement the empty network as a baseline, but add three other examples in order to study different network structures (see Figure C.1). Comparing the networks in Study I with those in Study II, both studies implement the empty network (in which information transmission is precluded), a weakly balanced network (in which $\hat{\sigma}$ is an inefficient equilibrium), and the star network (in which $\hat{\sigma}$ is not an equilibrium). While Study I accompanies the weakly balanced network with a strongly balanced network to have an example in which $\hat{\sigma}$ is efficient, Study II accompanies the star network with another unbalanced network, which we call the *unbalanced network* that features different sender degrees within one treatment.

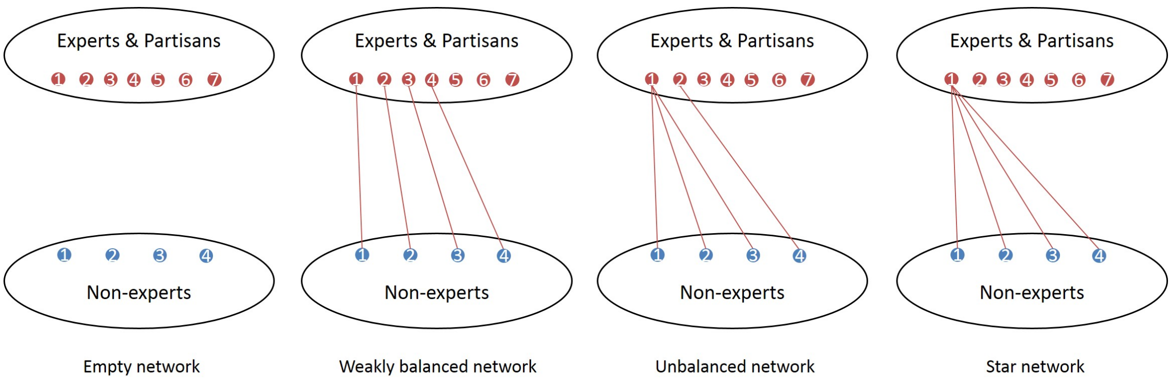


Figure C.1: The four treatments in Study II. The empty network is the baseline treatment. The sincere strategy profile $\hat{\sigma}$ is an inefficient equilibrium in the weakly balanced network; it is not an equilibrium in the unbalanced and the star network. LTED σ^* is an efficient equilibrium in any network.

In the working paper (Buechel and Mechtenberg, 2017), we report the results of both Study I and Study II in parallel and observe that they lead to the same conclusions. There are two interesting differences between the results of the two studies that we recap here since we refer to these differences in section 5 when addressing misinformation:

(1) A part of Result 2 ("Experts significantly more often play sincere [...] in the weakly

[13] In the current version, Proposition 2.3 is richer than it was in this earlier working paper version.

balanced than in the star network.") is not confirmed in Study II. This is revealed by Tables C.3.1 and C.3.2 below.

(2) Results 4 and 5 about the relative inefficiency of the star network are stronger in Study II. This is revealed by Tables C.3.3, C.3.4, and C.3.5 below.

These two observations suggest that the presence of troll senders (1) makes human senders prone to pass on their signal to their audience regardless of the network structure, and (2) makes the star network even less efficient, compared to balanced networks, mostly due to followers of a troll sender in the center of the star.

Table C.3.1. Behavior of experts

	vote signal	vote opposite	send signal	send opposite	sincere
empty	560	21	-	-	*560*
($N = 600$)	93.3%	3.5%	-	-	*93.3%*
weakly balanced	550	31	309	15	530
($N = 600$)	91.7%	5.2%	89.1%	4.3%	88.3%
unbalanced	552	22	158	4	534
($N = 600$)	92.0%	3.7%	88.8%	2.3%	89.0%
star	556	27	76	1	550
($N = 600$)	92.7%	4.5%	91.6%	1.2%	91.7%
Total	2,218	101	543	20	2,174
($N = 2,400$)	92.4%	4.2%	89.3%	3.3%	90.6%

Table C.3.1: Behavior of experts by treatment in Study II. The action 'vote (send) opposite' means vote (send message) A when signal is B^* and vice versa. In addition to the displayed categories 'vote signal' and 'vote opposite' experts could abstain. In addition to the displayed categories 'send signal' and 'send opposite' experts could send an empty message. Experts without an audience are sincere if they vote their signal. Experts with an audience are sincere if they vote their signal and also send it. This table corresponds to Table 6 in Appendix A.1.

Table C.3.2. Sincere senders

	Logit 1: Send Signal		**Logit 2: Sincere**	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
unbalanced	-0.029	(0.264)	0.080	(0.301)
star	0.289	(0.356)	0.506	(0.366)
Intercept	2.096***	(0.320)	1.878***	(0.235)
N	608		608	
Log-likelihood	-206.44		-226.34	
Wald $\chi^2_{(2)}$	1.49		6.95	
p-value Wald test	0.475		0.031	

$^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$

Table C.3.2: Estimation results for Study II: Logistic regression sincere senders by treatment. Senders are experts with at least one link. Dependent variable in Model 1 is 'send signal,' which is 1 if the expert's message equals her signal (and zero otherwise). Dependent variable in Model 2 is sincere behavior, which equals 1 if sender both sends and votes her signal. Robust standard errors in parentheses adjusted for sessions. Baseline category is the weakly balanced network. Observe that Model 1 is not well-specified according to Wald test. This table corresponds to Table 7 in Appendix A.1.

Table C.3.3. Fisher exact tests on efficiency

	weakly balanced	unbalanced	star
empty	0.323	0.022	0.002
weakly balanced		0.219	0.007
unbalanced			0.244

Table C.3.3: p-values of Fisher exact tests comparing efficiency of two treatments in Study II. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses. This table corresponds to Table 11 in Appendix A.1.

Table C.3.4. Dependent variable: Efficiency

	ologit 1		**ologit 2**	
Variable	Coeff.	(Std. Err.)	Coeff.	(Std. Err.)
empty			-0.059	(0.140)
weakly balanced	0.059	(0.140)		
unbalanced	-0.276*	(0.164)	-0.335	(0.210)
star	-0.711**	(0.319)	-0.770***	(0.243)
uniform signal	2.027***	(0.135)	2.027***	(0.135)
Intercept cut 1	-1.611	(0.208)	-1.670	(0.251)
Intercept cut 2	-0.572	(0.122)	-0.631	(0.179)
N	800		800	
Log-likelihood	-513.262		-513.262	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table C.3.4: Estimation results for Study II: Ordered logit. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses. Robust standard errors in parentheses adjusted for sessions. Less clusters than parameters simply mean that joint significance (Wald test) cannot be tested. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category. This table corresponds to Table 12 in Appendix A.1.

Table C.3.5. Dependent variable: Expected payoff EP

	OLS 1		**OLS 2**	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
empty			0.754	(0.796)
weakly balanced	-0.754	(0.796)		
unbalanced	-3.528**	(0.997)	-2.773**	(0.915)
star	-7.703*	(2.847)	-6.949**	(2.219)
uniform signal	31.214***	(0.844)	31.214***	(0.844)
Intercept	64.411***	(1.312)	63.656***	(1.625)
N	800		800	
R^2	0.347		0.347	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table C.3.5: Estimation results for Study II: OLS with expected payoff EP as dependent variable. Robust standard errors in parentheses adjusted for sessions. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category. This table corresponds to Table C.1.3 in Appendix C.1.

References

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